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**Mathematics Behind Artificial Intelligence :A review**

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**Abstract**

In this paper, a brief review on the aspect of how mathematical tools backed the creation and growth of Artificial Intelligence (AI) is presented by focusing and highlighting the following issues:

Starting with the explanation of the term “Artificial Intelligence” , the chronological developments of AI are presented from ancient Greek mythological philosophy to the 21<sup>st</sup> century. The paper then highlights how the different mathematical theorems and tools derived from various branches of Mathematics, Viz. Differential calculus, Multivariable Calculus, Integral calculus, Error minimization, Optimization via gradient descent, Advanced logistic Regression, help in Robotic navigation. Another objective of the review work is to focus the involvement of Discrete random variables, Continuous random variables, Bayes formula and Normalization, Topology, Game theory, Graph theory, Linear transformation, Eigenvalues and Eigenvectors, Neural network, Three Layers: Input layers, Hidden layers, Output Layers, Tensor, Vector, Differential equations, Fourier transform, Laplace Transform, Hankel Transform, Mellin Transform, Hilbert Transform, Z-Transform in AI architecture, its design and in working. Finally, the relevant applications of AI , and popular software such as ChatGPT, Google Gemini, Claude, Microsoft 365 Copilot, You.com, Brave Leo, GitHub Copilot, Perplexity AI, Canvas AI, Jasper, Google Veo, etc., are brought in to discussion .



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## **1.0 Introduction:**

Artificial Intelligence (AI) revolutionizes in the fields of modern society, human civilization science and technology by systemizing complex scenarios, steering up massive datasets for hidden patterns, increasing research activities, optimizing complex simulations (like climate or drug discovery), accelerating medical studies and autonomous systems to cybersecurity, acting as a powerful tool to augment human intelligence (??) and enabling the solution of complex problems. It enhances efficiency, improves decision-making, and fosters new discoveries across nearly all scientific and technological domains. Human civilization is at climax position due to unprecedented applications of AI, Machine Learning , Deep Learning, Data Science, Optimization Technique, and Powerful Simulation Theory .

To be precise, Artificial intelligence is a subarea of computer science and technology and focuses on creating intelligent machines capable of performing tasks just like human beings. These tasks include problem-solving, speech recognition, and decision-making, and many more. More specifically, in AI, programs are developed to perform specific tasks, that is being utilized for a wide range of activities including remote sensing, electronic trading platforms, robot control, and medical diagnosis [Akerkar 2014; David 2024; Raissi et al., 2019; Berner et al., 2025; Kanvaria and Michael T Suraj, 2024] The formal study of artificial intelligence (AI) began in the mid-20th century, when the Dartmouth Workshop in 1956 was held in UK as its foundational moment.

The write up is subdivided in to two basic sections. The first section describes the chronological development of AI and its various uses in different fields, and Section two highlights significant applications of Mathematics in AI.



## 2. Analysis of the problem and Description:

### 2.1 Section one:

#### 2.1.1 Development of Artificial Intelligence:

[Akerkar. 2014;Waldehen, et al., 2021; Han et al., 2018; Weinan and Yu.2018]

In Greek mythology, King Minos of Crete manufactured Talos, a giant automaton made of bronze, that worked just like a God of metalworking. The main duty of this Giant was to protect the island from invaders by throwing very big stones at approaching ships. There are many more mythological stories on ‘how a machine could work just like a human being’. The chronological order of development of AI is as follows:

1921: Czech playwright Karel Čapek created a science fiction play entitled “[Rossum’s Universal Robots](#)” which laid the basic idea of “artificial people”. The artificial people were introduced as robots. This was the first known use of the word-‘Robot’-.

1929: Japanese professor Makoto Nishimura built the first Japanese robot, known as [Gakutensoku](#).

1949: Computer scientist Edmund Callis Berkley published the book “[Giant Brains, or Machines that Think](#)” which revolutionized the newer models of computers working just like human brains.

1950: Alan Turing published “[Computer Machinery and Intelligence](#)” which evolved a test of machine intelligence named as The Imitation Game.

1952: Arthur Samuel, a computer scientist composed a [program to play checkers](#), which is the first to ever learn the game in an independent way .

1955: John McCarthy organized a workshop at Dartmouth on “[artificial intelligence](#)” , and since then rigorous research has been going on to advance the science and technology .

1958: John McCarthy developed a programming language known as [LISP](#) (acronym for List Processing), for AI research, which is still in popular use in the modern time .

1959: Arthur Samuel initiated the term “[machine learning](#)” for the first time when performing a speech about teaching machines how to play chess much better than the humans who can programme them.



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1961: The first industrial robot [Unimate](#) was created in order to work on an assembly line at General Motors in New Jersey, tasked with transporting die casings and welding parts on cars (which was seemed to be very risky for human beings).

1965: Two scientists named Edward Feigenbaum and Joshua Lederberg developed [the first “expert system”](#) – an AI program to replicate the thinking and decision-making abilities of human experts.

1966: Joseph Weizenbaum developed the first “chatterbot” (later shortened to chatbot), [ELIZA, a mock psychotherapist](#), that facilitated natural language processing (NLP) to interact with humans.

1968: Soviet mathematician Alexey Ivakhnenko published an article entitled “Group Method of Data Handling” in the journal “Avtomatika,” which developed an innovative approach to AI that becomes very powerful for scientific research . This new tool is known as “Deep Learning.”

1979: The American Association of Artificial Intelligence which is now known as the [Association for the Advancement of Artificial Intelligence](#) (AAAI) was founded and a continuous progress is going on .

2006: Companies such as Twitter, Facebook, and Netflix started utilizing AI as a part of their advertising and user experience (UX) algorithms.

2011: Apple released Siri, the first popular virtual assistant.

2026: The current significance of Artificial Intelligence (AI) development, specially moving into 2026, is magnified by a shift from experimental generative models to **agentic AI**, which focuses on autonomous, goal-driven actions rather than just content creation. Agentic AI refers to intelligent systems that move beyond passive, generative responses to actively plan, reason, and autonomously execute multi-step tasks to achieve specific goals with minimal human oversight. Unlike conventional AI that reacts to prompts, agentic AI acts as an autonomous agent, using tools, memory, and cognitive skills to interact with digital environments and solve complex problems.

Some Popular Software for mathematics research under AI are:

ChatGPT, Google Gemini, Claude, Microsoft 365 Copilot, You.Com, Brave Leo, GitHub Copilot, Perplexity AI, Canva AI, Jasper, Google Veo , DUG Insight, Decide IT, Physitrack, Physibel Software Suite, etc,

A typical working model of a Robot is displayed in Figure 1



Figure 1: A Typical Robot capable of working like a human being

### *2.1,2 .Principal components of AI and basic computational platforms*

AI is an interdisciplinary science with many approaches. Some of the most relevant components are highlighted below: [2,3,4,6][ David, 2024; Kutyniok 2022; Waldehen et al, 2021; Han et al., 2018]

**Computer vision** is a field of artificial intelligence (AI) that enables machines to understand, analyze, and interpret visual data from the real world, such as images and videos, mimicking human sight. In general, Computer vision transforms visual input (pixels) into numerical data, allowing algorithms to detect edges, shapes, and textures to classify or recognize content. Computer vision relies on machine learning and particularly deep learning (convolutional neural networks) to train models to identify patterns with high accuracy.

**Cognitive computing** simulates human thought processes to solve complex, ambiguous problems by learning, reasoning, and understanding language. It uses neural networks, NLP, and machine learning to analyze massive unstructured datasets (text, images, audio) to support human decision-making rather than fully automating it.

**Machine learning** is a subset of artificial intelligence (AI) that enables systems to learn from data, identify patterns, and make decisions without explicit programming. It uses algorithms to analyze large datasets and improve performance over time, powering technologies like recommendations, speech recognition, and generative AI.

**Deep learning** is a subset of AI and machine learning that uses multi-layered artificial neural networks to simulate the human brain's decision-making process. It processes large datasets to identify complex patterns, improving accuracy automatically over time for tasks like image recognition, natural language processing, and autonomous driving. It has four important components :

Figure 2 gives the four component structure of AI

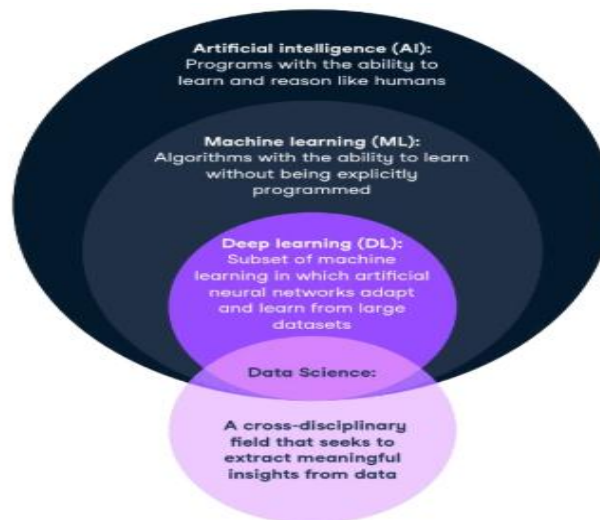


Figure 2 : Artificial intelligence : the four components

### 2.1.3 AI in relation to data science and other key concepts. Source: [DataCamp](#)

**Neural Network Structure:** Deep learning models are "deep" because they contain many layers—often 4 or more—between input and output, which process data with increasing levels of abstraction. Neural networks allow computers to learn from experience and understand the world in terms of a hierarchy of concepts.



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Thanks to neural networks, researchers have been able to solve some of the most complex problems, including image and video processing. Equally, a type of neural network called the transformer is key to understanding the development and rise of generative AI.

**Learning Process:** These systems learn from data by continuously adjusting the strength of connections (weights and biases) between nodes in the network through methods like back propagation.

**Data Volume:** Unlike traditional machine learning, which may plateau, deep learning performance generally improves as the volume of training data increases.

**Unstructured Data:** Deep learning excels at analyzing unstructured data, such as images, audio, and text, which is difficult for traditional algorithms to process.

**Natural language processing (NLP):** A field of AI that focuses on the interaction between computers and humans through natural language. The ultimate objective of NLP is to enable computers to understand, interpret, and generate human languages in a way that is both meaningful and useful.

**Robotics :** AI is key to the development of robotics. AI technologies can be integrated into robotic systems to enhance their capabilities and enable them to perform more complex tasks. AI in robotics allows robots to learn from experience, adapt to new situations, and make decisions based on data from sensors.

#### *2.1.4 Basic working modules of AI*

How AI works [Waldehen, et al., 2021; Berner et al., 2025; Weinan and Yu, 20018; Kanvaria and Michael T Suraj 2025 ]

In order to describe the essence and a full understanding of AI fundamentals, it's helpful to understand the steps that go into making an AI system function. The followings are worth-mentioning :

**Data collection.** The first step in any AI project is gathering data. This could be anything from pictures and text to more complex data like human behavior. The data serves as the raw material from which the AI system will learn.

**Data preparation.** Once the data is collected, it needs to be prepared and cleaned. This means removing any irrelevant information and converting the data into a format that the AI system can understand.



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**Choosing an algorithm.** An algorithm is like a recipe for how the AI system will process the data. Different algorithms are better suited for different tasks. For example, we might use a specific algorithm for image recognition and another for natural language processing. We can explore various types of algorithms in a separate article.

**Training the model.** The prepared data is fed into the chosen algorithm to "train" the AI model. During this phase, the model learns to make predictions or decisions based on the data.

**Testing the model:** After training, the model is tested to see how well it performs. If it's not accurate enough, it may need to be trained further or adjusted.

**Deployment.** Once the model is trained and tested, it's ready to be deployed into a real-world application. This could be anything from a chatbot answering customer queries to a medical AI analyzing X-rays.

**Ongoing learning.** Many modern AI systems can learn and adapt over time. This means they can improve their performance as they gather more data, making them more efficient and accurate.

### *2.1.5 Applications of AI: A few examples;*

[Waldehen et al., 2021; Weinan and Yu. 2018; Kanvaria and Michael T Suraj 2024]

The possibilities of AI are endless, and it's not surprising that companies from all kinds of sectors and industries are integrating AI technologies to optimize their business processes and support decision-making. Here are some ways in which artificial intelligence is used in today's world:

#### **AI in software development:**

Next-generation AI tools already offer many opportunities in software development. The number of use cases is rapidly growing as new AI-powered tools reach the market and developers become familiar with them. Today, coders can use AI for tasks like task automation, bug filing, testing processes, code translation, and code optimization.

#### **AI in data analytics**

The same goes for data analytics. With the rise of machine learning and generative AI tools like ChatGPT,



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data professionals can rapidly conduct data processing tasks, create compelling visualizations in a matter of clicks, or set the coding foundations for powerful predictive models.

### **AI in cyber security**

AI is rapidly revolutionizing the field of cybersecurity. AI-driven tools can help spot security threats, implement automated responses, and advance mitigation strategies. As Brian Murphy, CEO of ReliaQuest, puts it in our DataFramed podcast on How AI is Changing Cybersecurity. AI is a useful tool to help make security possible. It helps us increase visibility, get access to data in different data types and different table fields and different tables and put like next to like so we can make accurate decisions with accurate security information.

### **AI in cloud computing**

It's sometimes hard to draw a clear line between cloud computing and AI, for nearly all operations hosted in the cloud are powered by AI technologies. From resource allocation optimization and cost management to service delivery and cloud security, AI is at the core of the cloud business.

### **AI in finance**

AI in finance is rapidly transforming the banking processes to make them much more efficient and also cost-effective. Through the examination of vast data sets, AI algorithms can automate manual tasks, freeing up employees to focus on higher-value work. AI is also enhancing fraud detection and prevention. By leveraging data in finance, machine learning models can analyze millions of transactions to detect subtle patterns indicating any fraud faster and also more accurately than humans. AI in banking also uses these analyses to catch fraudulent transactions in real time, reducing fraud losses.

### **AI in healthcare**

There are many applications of AI in healthcare, from disease detection (for example, algorithms can analyze medical images to identify early signs of diseases like cancer) and drug discovery to patient monitoring and virtual nursing. In the future, AI will likely be even more embedded in healthcare. It will provide real-time support, advanced diagnostics, and highly personalized treatment recommendations. With AI, healthcare can become more proactive, continuously monitoring and predicting health issues before they escalate, leading to a more responsive and efficient system.



### 2.1.6 AI's role in modern society:

[Akerkar 2014; David 2024; Han et al., 2018; Berner et al., 2025; Weinan and Yu 2028]

The main role of AI is to boost efficiency, automate tasks, enhance decision-making, and solve complex problems across industries like healthcare, finance, and transport, but it also presents challenges in ethics, bias, privacy, and job displacement, requiring responsible development to balance progress with societal well-being. It integrates into daily life through personal assistants, personalized recommendations, and advanced data analysis, transforming how we work, consume media, and interact with technology. A few significant Roles & Impacts are :

#### **Boosting Productivity & Efficiency:**

AI automates repetitive tasks, analyzes massive datasets quickly, and optimizes operations in manufacturing, logistics, and customer service.

**Transportation:** Powers self-driving cars, optimizes traffic flow, and improves logistics.

**Education:** Offers personalized learning experiences and administrative automation.

#### **Solving Grand Challenges:**

Assists in climate research, predicts disease outbreaks, and improves disaster response. **Improving Daily Life:** Powers smartphone assistants (Siri, Google Assistant), media recommendations (Netflix, Amazon), and personalized user experiences.

#### **Increasing Safety & Security:**

Enhances surveillance, detects threats in cybersecurity, and aims to reduce accidents in autonomous vehicles.

### 2.1.7 AI, Main Roles in Science & Research

[Raissi et al., 2019; Weinan and Yu 2018; Kanvaria and Michael T Suraj. 2024]

**Data Analysis:** Quickly processes vast scientific data (genomics, astronomy) to find patterns beyond human capability, leading to new discoveries.



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**Complex Simulations:** Models complex systems like climate, molecular interactions, and materials, predicting outcomes before experiments.

**Accelerated Discovery:** Speeds up drug discovery, materials science, and even mathematical proofs by generating hypotheses and identifying relationships.

**Automation:** Automates repetitive lab tasks, freeing researchers for more creative work.

**Scientific Literature:** Systematically interrogates scientific papers to derive complex ideas and relationships.

**In brief, the roles of Robotics in Technology & Innovation are :**

- Automation & Efficiency: Powers robotics, optimizes energy grids, and streamlines manufacturing.
- Personalization: Tailors experiences in healthcare, e-commerce, and entertainment.
- Cybersecurity: Detects threats, prevents fraud, and enhances data security through pattern recognition.
- Autonomous Systems: Drives self-driving cars and smart devices.
- Human-AI Collaboration: Creates more efficient teams and requires new skills, changing programming and tech roles.
- Problem Solving: Helps address global challenges like climate change and disease prediction.
- Ethical Considerations: Necessitates responsible development to manage bias, privacy risks, and unintended consequences.
- Innovation Driver: Enables development of new algorithms, hardware, and computing paradigms, shaping a data-driven future.



## 2.2 Section two

### 2.2.1 Key mathematical fields and their applications in AI :

[Kutyoniok 2022, Raissi et al., 2019; Berner et al 2025; Kanvaria and Michael T Suraj, 2024]

#### 2.2.1.1 *Linear Algebra (Data Representation):*

**Data Structures:** Uses vectors, matrices, and tensors (multi-dimensional arrays) to represent and manipulate data.

**Deep Learning:** Neural networks rely on matrix multiplication for forward and backward propagation.

**Dimensionality Reduction:** Techniques like Principal Component Analysis (PCA) use eigenvalues and eigenvectors to simplify complex datasets.

**Image Processing:** Pixels are treated as matrices, allowing for object recognition.

#### **Calculus (Optimization):**

**Gradient Descent:** Functions are optimized by finding derivatives, minimizing errors, and improving model accuracy.

**Model Training:** Essential for understanding how AI models change and make predictions.

#### **Probability and Statistics (Handling Uncertainty):**

**Prediction and Inference:** Helps AI make informed decisions despite incomplete data.

**Algorithms:** Powers Bayesian networks, hidden Markov models, and Gaussian mixture models.

**Data Analysis:** Used for understanding data distributions, hypothesis testing, and fraud detection.

#### 2.2.1.2 *Optimization Techniques:*

**Algorithm Efficiency:** Methods like stochastic gradient descent train models faster.

**Support Vector Machines (SVM):** A mathematical framework for classification and regression tasks.

#### **Other Mathematical Areas:**

**Graph Theory:** Used for modeling relationships in network analysis.

**Information Theory:** Helps in understanding data entropy and optimizing information transmission.



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These mathematical tools allow AI to move from raw data to intelligent actions, enabling applications like self-driving cars, chatbots, and recommendation systems. The world of AI research is constantly evolving, and mathematicians are at the forefront of these advancements.

### 2.2.3 The exciting areas where math is shaping the future of AI:

**Explainable AI (XAI):** As AI models become more complex, understanding how they arrive at decisions becomes crucial. Mathematicians are developing techniques based on game theory and logic to make AI models more transparent and interpretable.

**Natural Language Processing (NLP):** Enabling computers to understand and generate human language requires advanced mathematical techniques. Topics like information theory and graph theory are being used to model the complexities of human language and enable machines to have meaningful conversations.

**What about the Future of Maths** As we continue to unravel the mysteries of AI and its applications, the profound nexus between mathematics and artificial intelligence will undoubtedly remain at the forefront of technological advancement, shaping the future of our digital world. Embracing this synergy opens doors to unprecedented possibilities, where the art of mathematics converges with the science of AI, illuminating new horizons of discovery and innovation.

### 2.2.4 Challenges & Considerations:

**Job Displacement:** Automation can replace some jobs, necessitating workforce adaptation.

**Ethical Concerns:** Issues around algorithmic bias, data privacy, surveillance, and accountability are significant.

**Building Trustworthy AI:** Ensuring fairness, transparency, and alignment with human values is crucial. In essence, AI acts as a powerful tool, augmenting human capabilities and driving innovation, while also demanding careful management of its societal implications.



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*Note: The materials of this paper are collected from various resources like books, research papers, internet, lecture notes etc*