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Application of Statistical Probability in Estimating Expected Number of Rainy Days

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Abstract

The concept of statistical probability, introduced by von Mises, has been applied in defining the probability of occurrence of rainfall in terms of rainy days together with the defining mathematical expectation of number of rainy days on the basis of the data on already happened outcomes. In order to show the application of the two definitions, probabilities of occurrences of different numbers of rainy days in each of the 12 months at Mumbai as well as the respective expected numbers of rainy days have been estimated by these two definitions.

Key Words: Natural outcomes, probability, statistical definition, mathematical expectation, rainy day

1. Introduction:

Probability is a basic statistical tool for understanding and explaining of various phenomena in almost every branch of science [Bartlet (1962) , Laplace (1954) , Papoulis (1965) , Mckay (2019)]. The history of development of probability can be broadly classified into five broad stages in terms of eras as follows [Chakrabarty (2014) , Maistrov (1974)]:

(1) The Era of Prehistory of Probability Theory, whose beginning was lost in the dust of antiquity.

(2) The Era of Origin of Probability Theory as a Science, continued from the middle of the seventeenth century up to the beginning of the eighteenth century.

(3) The Bernoullian Era of Probability Theory, began with the appearance of the treatise "Arts Conjectandi", by James Bernoull in 1713 and continued up to the early period of nineteenth century.

(4) The Era of Probability Theory in Russian School that continued from the middle of the nineteenth century up to the first quarter of the 20th century mainly with the works of Chebyshev, Markov and Liapounov.

(5) The Era of Modern Probability Theory, began with the formulations of axioms mainly by Bernstein and by Kolmogorov independently.

The theory of probability has come to the current stage of development through the following five approaches

(1) Subjective Approach introduced by Thomas Bayes [Bayes (1938)],

(2) Intuitive Approach due to Koopman & Savage [Koopman (1940*a* 1940*b*) , Savage (1954 , !961)],

(3) Classical Approach due to Bernoull [Bernoulli (1713) , Chakrabarty (2005 , 2006 , 2009)],

(4) Empirical Approach, introduced mainly by von Mises also knows as relative frequency approach as well as statistical approach [Camp)1962) , Feller (1968) , Fisher (1930) , von Mises (1939 , 1941)],

(5) Axiomatic Approach due to Bernstein & Kolmogorov [Berstein (1927 , 1946) , Jack & Albert (1978) , Kolmogorov (1933, 1956)]

and (6) Theoretical Approach, introduced by Chakrabarty during the first decade of the 21st century [Chakrabarty (2004 , 2007 , 2008 , 2009 , 2010*a* , 2010*b* , 2010*c* , 2011)].

In every approach, as mentioned above, probability is defined or determined on the basis of random experiment either performing the actual experimentation or prior to performing it. In reality however, we observe phenomena which are not to be performed but happened automatically and which consist of several possible outcomes [Isaac , Joy Zuonaki (2010) , Papoulis (1965), Chakrabarty (2011)]. In this case, we don't have scope of performing experimentation. What we can do; we can simply collect data from the already happened outcomes of the experiment. Probability of occurrence of some event may be required to be calculated in such cases. In this study, probability of occurrence of an event has been defined in such cases. Moreover, mathematical expectation of associated random variable has also been defined in the same case.

Central tendency [Chakrabarty (2015*a* , 2015*b* , 2020*b* , 2021*a* , 2021*b*, 2021*e* , 2022*c* , 2022*d* , 2022*e* , 2022*f* , 2023) , Weisberg (1992) , Williams (1984)] is one of the basic characteristics of data which plays a vital role in statistical analysis of data. A number of formulations, though may not be as sufficient as to handle all the real situation, have already been developed for measuring central tendency of data [Chakrabarty (2021a , 2021b, 2021e , 2021f , 2022a , 2022b) , Williams (1984)] which is basically based on measures of average [Chakrabarty (2016 , 2017 , 2018b , 2018c , 2018d , 2018e , 2018g , 2019b , 2019c , 2020a , 2021c , 2021d , 2021g]. There had already been several studies on various aspects like measures of characteristics of rainfall, trend of rainfall, forecasting on rainfall etc. [Basak & Sahu 2019) , Chakrabarty (2014 , 2021h , 2021i) , Hills (2015) , Jose, Tercio, Lincoln et al., (2020) , Krishnakumar, Prasada et al.,m (2009) , Isaac , Joy Zuonaki (2010) , Nikumbh, Chakraborty & Bhat (2019) , Taxak , Murumkar & Arya (2014)]. The studies on rainfall done so far are mostly based on non-probabilistic approach. Study on rainfall has hardly been done so far by probabilistic approach which has been applied in the current study on the same. The concept of empirical probability has been applied in defining the probability of occurrence of rainfall in terms of rainy days together with the definition of mathematical expectation of number of rainy days on the basis of the data on already happened outcomes. In order to show the application of the two definitions, probabilities of occurrences of different numbers of rainy days in each of the 12 months at Mumbai as well as the respective expected numbers of rainy days have been estimated by these two definitions.

2. Rainy Days – Probability & Mathematical Expectation:

Probability of Number of Rainy Days

Probability was statistically defined by von Mises in the following two ways [von Mises (1939 , 1941)]:

Definiton (1): If a trial is repeated N times under identical condition and if out of the N repetitions an event E occurs n times then the probability of occurrence of the event E, denoted by $P(E)$, is a number towards which the ratio $\frac{n}{N}$ approaches as N becomes larger i.e.

$$\frac{n}{N} \rightarrow P(E) \quad \text{as} \quad N \rightarrow \infty$$

i.e. $P(E)$ is the limiting value of $\frac{n}{N}$ as N becomes larger and larger.

Definition (2): If a trial is repeated N times under identical condition and if out of the N repetitions an event E occurs n times then the probability of occurrence of the event E, denoted by $P(E)$, is a number such that the number of occurrence of the event E approaches $N.P(E)$ as N becomes larger i.e.

$$n \rightarrow N.P(E) \quad \text{as} \quad N \rightarrow \infty$$

This definition is just the inverse versions of **Definition (1)**.

This definition states that the number of occurrence of the event E out of N repetitions of the trial can be approximated by $N.P(E)$ provided N is large [Fisher (1930) , Papoulis (1965) , Papoulis & Pillai (2002)].

Some Properties:

The following fundamental properties of probability can be obtained from the definition:

(1) For any event E, $0 \leq P(E) \leq 1$.

$P(E) = 0$ iff E is impossible event $P(E) = 1$ iff E is certain event.

(2) Probability of non-occurrence of the event is $1 \quad P(E)$.

(3) If A and B are mutually exclusive then the probability of occurrence of either A and B is the sum of the individual probabilities of occurrences of A and B

i.e. $P(A \cup B) = P(A) + P(B)$

In general, if E_1 , E_2 , ………… , E_n are mutually exclusive events then the probability of occurrence of either any of the n events is the sum of their individual probabilities of occurrences

i.e. $P(E_1 \cup E_2 \cup \ldots\ldots\ldots \cup E_n) = P(E_1) + P(E_2) + \ldots\ldots\ldots + P(E_n)$

Suppose that E is an event that denotes occurrence of r rainy days in a month.

Consider the observations on happenings of rainfall in the month on a number of years (say N years) i.e. on N repetitions of the happenings.

Since the phenomenon has happened naturally, it is free from error that occurs due to performing of experiment.

Moreover, the natural happening of the phenomenon can be thought of as the performing of experiment on rainfall not by human but by nature.

If out of N repetitions the event E occurs $N(E)$ times then the probability of occurrence of the event E, denoted by $P(E)$, can be defined by the number towards which the ratio $\frac{n}{N}$ approaches as N becomes larger i.e.

$$\frac{n}{N} \rightarrow P(E) \quad \text{as} \quad N \rightarrow \infty$$

i.e. $P(E)$ is the limiting value of $\frac{n}{N}$ as N becomes larger and larger.

Note:

For finite N i.e. for sample of finite size, the value of this ratio may not be equal to the actual value of the probability $P(E)$. However, it can be regarded as estimator of $P(E)$ due to the above limiting property [Papoulis & Pillai (2002)].

Some Results:

The following results can be obtained from the properties of probability as mentioned above:

(1) Estimate of probability of non-occurrence of the event E is

1 Estimate of probability of occurrence of E $P(E)$

i.e. 1 Estimate $P(E)$

(2) If A and B are mutually exclusive then estimate of probability of occurrence of either A and is the sum of the estimates of individual probabilities of occurrences of A and B.

(3) In general, if E_1 , E_2 , …………. , E_n are mutually exclusive events then estimate of probability of occurrence of either any of the n events is the sum of their estimated individual probabilities of occurrences.

Mathematical Expectation of Number of Rainy Days

In probability theory, the expected value (also called expectation, expectancy, expectation operator, mathematical expectation, mean, average, & first moment) is a generalization of the weighted average. Informally, the expected value is the arithmetic mean of a large number of independently selected outcomes of a random variable. The expected value of a random variable with a finite number of outcomes is a weighted average of all possible outcomes.

If a random variable X assumes the values

$$X_1, X_2, \dots\dots\dots, X_n$$

with respective probabilities

$$p_1, p_2, \dots\dots\dots, p_n$$

then the mathematical expectation of a random variable X is defined by

$$E(X) = \sum_{i=1}^{n} X_i \, P(X = X_i) = \sum_{i=1}^{n} p_i X_i$$

[Papoulis.& Pillai (2022) , Rose (2019) , Wasserman (2010)].

Accordingly, if

$$r_1, r_2, \dots\dots\dots, r_n$$

are the possible values of number of rainy days R occurring in a period with respective probabilities

$$p_1, p_2, \dots\dots\dots, p_n$$

then the mathematical expectation of the number of rainy days R in the period is defined by

$$E(R) = \sum_{i=1}^{n} r_i \, P(R = r_i) = \sum_{i=1}^{n} p_i r_i$$

Since

$$p_1, p_2, \dots\dots\dots, p_n$$

are not exact but approximate value, due to the limitation of number of observed data and since the variable R is non-negative integral valued, therefore, the nearest non-negative integral value of

$$\sum_{i=1}^{n} p_i r_i$$

will be estimated value of $E(R)$ (i.e. of expected number of rainy days).

Note:

In the case of interval values of number of rainy days, the mid points of the intervals are to be used in this formula. In this case, the interval which contains the value of

$$\sum_{i=1}^{n} p_i r_i$$

will be estimated interval value of $E(R)$.

3. Expectation of Rainy Days at Mumbai:

The definition of probability based on the data on already happened outcomes has been applied in estimating probability of occurrence of number of rainy days in different months at Mumbai. For this purpose, data on number of rainy days (month-wise) at Mumbai have been collected from the year 1969 onwards from Meteorological Department of Government of India and then the above formulation of probability has been applied in computing the desired values of probabilities.

The number of rainy days considered here are the point values

$$0 , 1 , 2 , 3 , 4 , 5$$

and the interval values

$$6 – 10 , 11 – 15 , 16 – 20 , 21 – 25 , 26 – 30$$

Estimated values of probabilities corresponding to these point/interval values of number of rainy days in different months, obtained by the formulation of probability defined above, have been shown in **Table – 5.1**. Estimated values of number of rainy days in different months, obtained by the formulation of mathematical expectation mentioned above, have been shown in **Table – 5.2**.

4. Result and Discussion:

If the probability of occurrence of zero rainy day at a place during a period is 1 then the period can be regarded as a certain non-rainy one.

In reality, there may be rainfall during a non-rainy period due to some random cause that occurs accidently but not regularly and not always so that 1 rainy day can occur during a non-rainy month with very small (near to 0) probability. Thus, if the probability of occurrence of zero rainy day during a period is not 1 but near to 1 and the probability of occurrence of 1 rainy day during the period is very small such that the probability of occurrence of either 0 rainy day or 1 rainy day is 1 (i.e. there are only 2 possible outcomes namely 0 and 1) then the period can be regarded as almost certain non-rainy period.

Similarly, if the probability of occurrence of 2 or more rainy days (i.e. at least 2 rainy days) is 1 then the period can be regarded as a certain rainy one.

Similarly, if the probability of occurrence of 2 or more rainy days is very near to 1 and the probability of occurrence of at least 1 rainy day is 1 then the period can be regarded as an almost certain rainy one.

In similar manner, the period can be regarded as more likely rainy period, equally likely rainy period or less likely rainy period depending upon the probability of occurrence of rainy days.

From the estimated probability distribution as shown in **Table – 5.1**, the following conclusion can be drawn for the station Mumbai:

1. No month is certain non-rainy.
2. The period January – April is almost certain non-rainy.
3. The period June – September is certain rainy.
4. The period October – December as well as the month May are more likely to be rainy.

Regarding estimated value of expected number of rainy days, it is to be number of rainy days is an integral value. Accordingly, from the computed values of expected number of rainy days as shown in **Table – 5.2**, the estimated values of the same obtained have been shown in the following table (**Table – 4.1**):

Table – 4.1

(Estimated expected number of rainy days at **Mumbai**)

Month	Number (or Interval of Numbers) of Rainy Days	Month	Number (or Interval of Numbers) of Rainy Days	Month	Number (or Interval of Numbers) of Rainy Days
January	0	May	1	September	15
February	0	June	14	October	4
March	0	July	23	November	1
April	0	August	22	December	0

Though the estimated value of number of rainy days in each of the 5 months

January , February , March , April & December

has been found to be 0, it does not imply that these months are certain to be completely free from influence of rainfall, Rainfall may occur in each of these 5 months though not every year.

From the estimated probability distribution, as shown in **Table – 5.1**, it is obtained that the number of occurrences of rainy days as follows:

(1) One rainy day is likely to occur in January once in 16 years.

(2) One rainy day is likely to occur in February once in 11 years.

(3) One rainy day is likely to occur in March once in 32 years.

(4) One rainy day is likely to occur in April once in 11 years.

(5) Occurrence of 1 or 2 rainy days is likely in December once in 5 years.

However, the following conclusions can be drawn from the numerical findings obtained from the study:

1. The months January, February, March, April & December are expected to be non-rainy ones.
2. The months May & November are expected to be almost non-rainy ones.
3. The month June is expected to be of moderate rainfall.
4. The months July, August & September are heavy rainy months.
5. The month October is expected to be of light rainfall.

It is to be noted that no month has been found to have 100% rainy days. On the other hand, no month is certain to be completely free from influence of rainfall. The maximum number of rainy days in a month has been found to be 28. This finding possibly carries some significance in terms of the maximum duration of continuous rainfall. Possibly, continuous rainfall at a place cannot last more than 28 days. This can be a hypothesis to be established or disestablished by the researchers.

Finally, one can conclude that the logical derivation of definition of probability based on data on already happened outcomes can be a useful statistical tool of analysis of data obtained from automatically happened or naturally happened events. Therefore, as per the meaning of research [Chakrabarty (2018*a* , 2018*f* , 2019*a*)], this extended definition of probability can be regarded as a fundamental research carrying significant potentiality of application in analysis of data. The concept of probability defined for already happened outcomes, introduced here logically, can be applied, in a similar manner, in estimating probability distribution and mathematical expectation of number of rainy days at other places of the globe also.

5. Tables of Findings – Values obtained from Computation:

Table – 5.1

(Estimated Probability Distribution of Number of Rainy Days at Mumbai)

Estimated probability distribution in January		Estimated probability distribution in February	
Number of Rainy Days	Estimated value of probability of occurrence	Number of Rainy Days	Estimated value of probability of occurrence
0	0.9375	0	0.90625
1	0.0625	1	0.09375
≥ 2	0	≥ 2	0

Table – 5.1: Continuation-1

(Estimated Probability Distribution of Number of Rainy Days at Mumbai)

Estimated probability distribution in March		Estimated probability distribution in April	
Number of Rainy Days	Estimated value of probability of occurrence	Number of Rainy Days	Estimated value of probability of occurrence
0	0.96875	0	0.90625
1	0.03125	1	0.09375
≥ 2	0	≥ 2	0

Table – 5.1: Continuation-2

(Estimated Probability Distribution of Number of Rainy Days at Mumbai)

Estimated probability distribution in May		Estimated probability distribution in June	
Number of Rainy Days	Estimated value of probability of occurrence	Number of Rainy Days	Estimated value of probability of occurrence
0	0.75	< 5	0
1	0.09375	5 – 9	0.1613
2	0.03125	10 – 14	0.4194
3 – 7	0.125	15 – 19	0.4194
> 7	0	> 19	0

Table – 5.1: Continuation-3

(Estimated Probability Distribution of Number of Rainy Days at Mumbai)

Estimated probability distribution in July		Estimated probability distribution in August	
Number of Rainy Days	Estimated value of probability of occurrence	Number of Rainy Days	Estimated value of probability of occurrence
< 16	0	< 12	0
16 – 20	0.3125	12 – 16	0.125
21 – 25	0.375	17 – 21	0.34375
26 – 28	0.3125	22 – 26	0.4375
> 28	0	27	0.09375
		> 27	0

Table – 5.1: Continuation-4

(Estimated Probability Distribution of Number of Rainy Days at Mumbai)

Estimated probability distribution in September		Estimated probability distribution in October	
Number of Rainy Days	Estimated value of probability of occurrence	Number of Rainy Days	Estimated value of probability of occurrence
< 5	0	0	0.1875
5 – 9	0.21875	1	0.0625
10 – 14	0.3125	2	0.15625
15 – 19	0.3125	3 – 7	0.5
20 – 24	0.09375	8 – 12	0.09375
25	0.0625	> 12	0
> 25	0		

Table – 5.1: Continuation-5

(Estimated Probability Distribution of Number of Rainy Days at Mumbai)

Estimated probability distribution in November		Estimated probability distribution in December	
Number of Rainy Days	Estimated value of probability of occurrence	Number of Rainy Days	Estimated value of probability of occurrence
0	0.625	0	0.78125
1	0.0625	1	0.125
2	0.1875	2	0.09375
3 – 7	0.125	> 2	0
> 7	0		

Table – 5.2

(Computed Value of Estimated Expected Number of Rainy Days at Mumbai)

Month	Computed value	Month	Computed value	Month	Computed value
January	0.0625	May	0.78125	September	14.21875
February	0.09375	June	13.29032	October	3.8125
March	0.03125	July	22.6875	November	1.0625
April	0.09375	August	21.3125	December	0.3125

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