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Observation Composed of a Parameter and Chance Error: An Analytical Method of Determining the Parameter

Dhritikesh Chakrabarty

Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India.
e-mail: dhritikesh.c@rediffmail.com, dhritikeshchakrabarty@gmail.com

Abstract

An analytical method has been developed to determine the true value of a parameter from its observations in the situation where the observations include chance errors but not any assignable error. The method was applied to find out the natural annual maximum and the natural annual minimum of ambient air temperature observed at Guwahati and the reliability of the method has been discussed.

Key Words: Parameter, chance error, observation, determination of parameter, analytical method.

1. Introduction

There are many situations where observations

$$X_1 , X_2 , \ldots\ldots\ldots\ldots , \quad X_n$$

are composed of some parameter and chance errors i.e.

$$X_i = \mu + \varepsilon_i \ , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , \quad n) \tag{1.1}$$

where (i) μ is the parameter

and (ii) ε_i is the chance error associated with X_i .

The existing methods of estimation, namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square provide

$$\bar{X} = \frac{1}{n}(\textstyle\sum_{i=1}^{n} X_i) \tag{1.2}$$

as estimator of the parameter μ . This estimator suffers from an error

$$\bar{\varepsilon_i} = \frac{1}{n}(\textstyle\sum_{i=1}^{n} \varepsilon_i) \tag{1.3}$$

which may not be zero [Allan (1962), Barnard (1949), Hald (1999), Ivory (1825), John (2000), Le Cam (1990), Kendall & Stuart (1977), Lehman & Casella (1998, Walker & Lev (1965)]. In other

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words, none of these methods can provide the true value of the parameter μ. In order to meet up this crisis, an analytical method has been developed for determining the true value of the parameter μ in such a situation. This paper is based on the development of this method and on one numerical application of the method in determining the annual maximum and the annual minimum of ambient air temperature at Guwahati.

The method developed is based on the theory of normal probability distribution discovered by a German mathematician *Carl Friedrich Gauss*, the credit for which discovery is also given by some authors to a French mathematician Abraham *De Moivre* who established that the normal distribution is an approximation to the binomial distribution discovered by *James Bernoulli* [Bernoulli (1713), Brye (1995), Chakrabarty (2005 , 2008)), De Moivre (1711 , 1718), Hazewinkel (2001), Marsagilla (2004), Stigler (1982), Walker (1985)]. This distribution is described by the probability density function

$$f(x : \mu , \sigma) = \{ \sigma (2\pi)^{½} \}^{-1} \; exp\, [- ½ \{(x - \mu)/\sigma\}^2] , \qquad (1.4)$$

$$- \infty < x < \infty , \; - \infty < \mu < \infty , \; 0 < \sigma < \infty .$$

where (i) X is the associated normal variable,

(ii) μ and σ are the two parameters of the distribution

and (iii) Mean of $X = \mu$ & Standard Deviation of $X = \sigma$.

The distribution is symmetric and satisfies the following properties:

If $X \sim N(\mu , \sigma)$, then

(i) $P(\mu - 1.96\, \sigma < X < \mu - 1.96\, \sigma) = 0.95$, (1.5)

(ii) $P(\mu - 2.58\, \sigma < X < \mu - 2.58\, \sigma) = 0.99$ (1.6)

& (iii) $P(\mu - 3\, \sigma < X < \mu - 3\, \sigma) = 0.9973$. (1.7)

2. Development of the Method

In the situation under consideration, the observations

$$X_1 , X_2 , \ldots\ldots\ldots\ldots , X_n$$

are such that they satisfy

$$X_i = \mu + \varepsilon_i \; , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n) \qquad (2.1)$$

were $\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n$ are values of the chance error ε associated with $X_1 , X_2 , \ldots\ldots\ldots\ldots , X_n$ respectively.

It is to be noted that

(1) $X_1 , X_2 , \ldots\ldots\ldots\ldots , X_n$ are known,

(2) $\mu , \varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots\ldots , \varepsilon_n$ are unknown

and (3) the number of linear equations in (2.1) is n with $(n + 1)$ unknowns implying that the equations are not solvable mathematically.

Reasonable facts /Assumptions regarding ε_i:

(1) $\varepsilon_1, \varepsilon_2, \ldots\ldots, \varepsilon_n$ are unknown values of the variable ε.

(2) The values of the chance errors $\varepsilon_1, \varepsilon_2, \ldots\ldots, \varepsilon_n$ are very small relative to the corresponding values of the variable X_i ie. $X_1, X_2, \ldots\ldots, X_n$.

(3) The variable ε_i assumes both positive and negative values.

(4) $P(-a - da < \varepsilon < -a) = P(a < \varepsilon < a + da)$ for every real a.

(5) $P(a < \varepsilon < a + da) > P(b < \varepsilon < b + db)$

& $P(-a - da < \varepsilon < -a) < P(-b - db < \varepsilon < -b)$

for every real positive $a < b$.

(6) The facts (3), (4) & (5) together imply that ε obeys the normal probability law.

(7) Sum of all possible values of each ε is 0 (zero) which together with the fact (6) implies that $E(\varepsilon) = 0$.

(8) Standard deviation of ε is unknown and small, say σ_ε.

(9) The facts (6), (7) & (8) together imply that ε obeys the normal probability law with mean (expectation) 0 and standard deviation σ_ε. Thus

$$\varepsilon \sim N(0, \sigma_\varepsilon) \qquad (2.2)$$

Note (2.1): Since $\varepsilon_1, \varepsilon_2, \ldots\ldots, \varepsilon_n$ are independently and identically distributed $N(0, \sigma_\varepsilon)$ variates, their mean defined by

$$\bar{\varepsilon}_i = \frac{1}{n}\left(\sum_{i=1}^{n} \varepsilon_i\right)$$

is a $N(0, \sigma_\varepsilon/\sqrt{n})$ variable.

2.1. The Method

Let the observations be arranged in ascending order of magnitude as –

$$X_{(1)} < X_{(2)} < \ldots\ldots < X_{(n)} \qquad (2.3)$$

From the model (2.1) satisfied by the observations,

$$X_{(i)} = \mu + \varepsilon_{(i)} \quad , \quad (i = 1, 2, \ldots\ldots, n) \qquad (2.4)$$

where $\varepsilon_{(1)} < \varepsilon_{(2)} < \ldots\ldots < \varepsilon_{(n)}$

which implies that $X_{(1)}$ contains the maximum negative error and $X_{(n)}$ contains the maximum positive error among the errors accompanied with the observations.

Now, let us construct the n averages defined by

$$\bar{X}_{(i)}(1) = \frac{1}{n-1} \sum_{j=1, j\neq i}^{n} X_{(j)}\,, \quad (i = 1, 2, \ldots\ldots\ldots, n) \tag{2.5}$$

Here, $$\bar{X}_{(1)}(1) > \bar{X}_{(2)}(1) > \ldots\ldots\ldots\ldots > \bar{X}_{(n-1)}(1) > \bar{X}_{(n)}(1) \tag{2.6}$$

From the model (2.1),

$$\bar{X}_{(i)}(1) = \mu + \bar{\varepsilon}_i (1) \tag{2.7}$$

where $$\bar{\varepsilon}_{(i)}(1) = \frac{1}{n-1} \sum_{j=1, j\neq i}^{n} \varepsilon_{(j)}\,, \quad (i = 1, 2, \ldots\ldots\ldots, n) \tag{2.8}$$

By **Note (2.1)**, some of the averages $\bar{\varepsilon}_{(1)}(1), \bar{\varepsilon}_{(2)}(1), \ldots\ldots\ldots\ldots, \bar{\varepsilon}_{(n-1)}(1), \bar{\varepsilon}_{(n)}(1)$ will lie above 0 and the others below 0. Consequently, some of the averages

$\bar{X}_{(1)}(1), \bar{X}_{(2)}(1), \ldots\ldots\ldots\ldots, \bar{X}_{(n-1)}(1), \bar{X}_{(n)}(1)$

will lie above μ and the others below μ.

Suppose, $\bar{X}_{(1)}(1), \bar{X}_{(2)}(1), \ldots\ldots\ldots\ldots, \bar{X}_{(k)}(1)$ fall above μ

and $\bar{X}_{(k+1)}(1), \bar{X}_{(k+2)}(1), \ldots\ldots\ldots\ldots, \bar{X}_{(n)}(1)$ fall below μ.

Then μ will lie within $\bar{X}_{(k+1)}(1)$ & $\bar{X}_{(k)}(1)$ with

$$\bar{X}_{(k+1)}(1) < \mu < \bar{X}_{(k)}(1) \tag{2.9}$$

Of course, it is trivial that

$$\bar{X}_{(n)}(1) < \mu < \bar{X}_{(1)}(1) \tag{2.10}$$

The inequality (2.9) can help to determine the true value of μ.

Note that positive error associated to $\bar{X}_{(i)}(1)$ decreases as i moves from 1 towards some point p and that negative error associated to $\bar{X}_{(i)}(1)$ decreases as i moves from n towards the point p. Thus, $\bar{X}_{(p)}(1)$ is the true value of μ. If the true value of μ cannot be determined clearly at this stage, one can exclude the two extreme observations, namely the 1st and the last ones and proceed with the same technique.

Now, excluding the two extreme observations namely $X_{(1)}$ and $X_{(n)}$ and retaining the remaining $(n-2)$ observations namely

$$X_{(2)} < X_{(3)} < \ldots\ldots\ldots\ldots < X_{(n-1)} \tag{2.11}$$

and then constructing the $(n-2)$ averages

$$\bar{X}_{(i)}(2) = \frac{1}{n-3} \sum_{j=2, j\neq i}^{n-1} X_{(j)}\,, \quad (i = 2, 3, \ldots\ldots\ldots, n\text{-}1) \tag{2.12}$$

such that $$\bar{X}_{(2)}(2) > \bar{X}_{(3)}(2) > \ldots\ldots\ldots > \bar{X}_{(n-2)}(2) > \bar{X}_{(n-1)}(2) \tag{2.13}$$

it is obtained that μ lies within $X_{(s+1)}(2)$ & $X_{(s)}(2)$ with

$$\bar{X}_{(s+1)}(2) < \mu < \bar{X}_{(s)}(2) \tag{2.14}$$

Of course, it is trivial that

$$\bar{X}_{(n-1)}(2) < \mu < \bar{X}_{(2)}(2) \qquad (2.15)$$

The inequality (2.14) can help, in the similar manner as in the earlier case, to determine the true value of μ. At this stage, if the true value of μ cannot be determined clearly, one can exclude the four extreme observations namely $X_{(1)}$, $X_{(2)}$, $X_{(n-1)}$ and $X_{(n)}$ and proceed with the same technique.

Excluding the four extreme observations namely $X_{(1)}$, $X_{(2)}$, $X_{(n-1)}$ and $X_{(n)}$ and retaining the remaining $(n - 4)$ observations namely

$$X_{(3)} < X_{(3)} < \ldots\ldots\ldots < X_{(n \square 2)} \qquad (2.16)$$

and then constructing the $(n - 4)$ averages

$$\bar{X}_{(i)}(3) = \frac{1}{n-5} \sum_{j=3, j \neq i}^{n-2} X_{(j)} , \quad (i = 3 , 4 , \ldots\ldots\ldots , n - 2) \qquad (2.17)$$

such that $\bar{X}_{(3)}(3) > \bar{X}_{(4)}(3) > \ldots\ldots\ldots > \bar{X}_{(n-3)}(3) > \bar{X}_{(n-2)}(3)$ (2.18)

it is obtained that μ lies within $X_{(p+1)}(3)$ & $X_{(p)}(3)$ with

$$\bar{X}_{(p+1)}(3) < \mu < \bar{X}_{(p)}(3) \qquad (2.19)$$

However in this case, it is trivial that

$$\bar{X}_{(n-2)}(3) < \mu < \bar{X}_{(3)}(3) \qquad (2.20)$$

This inequality can help, in the similar manner, to determine the true value of μ.

If the true value of μ cannot be determined clearly at this stage also, one can exclude the six extreme observations namely $X_{(1)}$, $X_{(2)}$, $X_{(3)}$, $X_{(n-2)}$, $X_{(n-1)}$ and $X_{(n)}$ and proceed with the same technique. The process can be continued further if necessary.

3. Analysis of Annual Extremes of Ambient Air Temperature

Temperature at a location attains at a maximum and at a minimum during the calendar year. Let T_1, T_2, …………, T_n be the observed values of the maximum temperature occurred at a location during the calendar years 1 , 2 , 3 , ……….., n respectively. The fluctuation of the value of T occurs primarily because of the changing geo-environmental condition of the place which is natural as well as the chance error associated with its measurement. If we consider that the geo-environmental condition remains same except the seasonal variation, the annual maximum or the annual minimum temperature also remain the same during the study period and the variation would be because of the chance error only. Thus, if β is the natural (corrected) annual maximum of temperature (*NAMaxT*) at the location,

$$T_i = \beta + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad (3.1)$$

where ε_i is the chance error associated to the observation T_i .

Similarly if

$$t_1 , t_2 , \ldots\ldots\ldots\ldots , t_n$$

be the observed values of the minimum temperature occurred at a location during the calendar years 1 , 2 , 3 , ……….., n respectively and α is the natural annual minimum of temperature (*NAMinT*) at the location,

$$t_i = \alpha + e_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n) \qquad (3.2)$$

where e_i is the chance error associated to the observation t_i .

Thus, the method discussed above can be suitably applied to determine the values of the two parameters α and β.

3.1. Determination of the *NAMaxT* at Guwahati:

Observed values of annual maximum Temperature at Guwahati observed during the period from 1969 to 2010 have been collected from the meteorological department of India [6 & 7]. These have been presented in **Table – 3.1.1** and arranged in ascending order of magnitude in **Table – 3.1.2**. Computations of the value of the ***NAMaxT*** based on (i) all observations, (ii) all observations excluding the two extreme observations namely the 1st & the 23rd ones, (iii) all observations excluding the four extreme observations namely the 1st, 2nd, 22nd & 23rd ones and (iv) all observations excluding the six extreme observations namely the 1st, 2nd, 3rd, 21st, 22nd & 23rd ones have been presented in **Table – 3.1.3**, **Table – 3.1.4**, **Table – 3.1.5** and **Table – 3.1.6** respectively.

Table – 3.1.1

Observed values of Annual Maximum Temperature at Guwahati (in Degree Celsius)

Year	Observed value	Year	Observed value	Year	Observed value	Year	Observed value
1969	37.1	1979	38.6	1989	36.7	2002	35.7
1970	36.6	1980	35.1	1990	36.0	2003	37.4
1971	36.0	1981	35.8	1991	37.4	2004	38.0
1972	35.7	1982	36.5	1992	39.4	2005	36.6
1973	39.0	1983	36.7	1993	36.4	2006	38.0
1974	36.1	1984	37.2	1994	37.3	2007	37.3
1975	39.2	1985	36.5	1995	36.3	2008	37.3
1976	39.0	1986	38.4	1996	37.2	2009	38.0
1977	35.3	1987	37.2	2000	37.5	2010	37.2
1978	36.8	1988	36.3	2001	36.7		

Table – 3.1.2

Observed values of Annual Maximum Temperature at Guwahati in ascending order (in Degree Celsius)

Serial No	Observed value	Serial No	Observed value	Serial No	Observed value	Serial No	Observed value
1	35.1	7	36.3	13	37.1	19	38.4
2	35.3	8	36.4	14	37.2	20	38.6
3	35.7	9	36.5	15	37.3	21	39.0
4	35.8	10	36.6	16	37.4	22	39.2
5	36.0	11	36.7	17	37.5	23	39.4
6	36.1	12	36.8	18	38.0		

3.1.1. Computation of the Value of *NAMaxT*:

Value of the *NAMaxT* at Guwahati has been determined from all the distinct observations and also from all the distinct observations excluding some extreme observations as shown below:

1. Value of *NAMaxT* based on all distinct observed values:

For determining the value of *NAMaxT*, one requires to construct the following table:

Table – 3.1.3

Mean of all observed values excluding the corresponding one (in Degree Celsius)

Serial No	Observed value	Mean	Serial No	Observed value	Mean	Serial No	Observed value	Mean
1	35.1	37.1500	9	36.5	37.0864	17	37.5	37.0409
2	35.3	37.1409	10	36.6	37.0818	18	38.0	37.0182
3	35.7	37.1227	11	36.7	37.0772	19	38.4	**37.0 ***
4	35.8	37.1182	12	36.8	37.0727	20	38.6	36.9909
5	36.0	37.1091	13	37.1	37.0591	21	39.0	36.9727
6	36.1	37.1045	14	37.2	37.0545	22	39.2	36.9636
7	36.3	37.0955	15	37.3	37.0500	23	39.4	36.9545
8	36.4	37.0909	16	37.4	37.0455			

It is observed that negative error associated to estimate is decreasing from the last estimate '36.9545' upwards and the positive error from the 1st estimate '37.15' downwards approaching point '37.0' where error is zero. This means, the value of *NAMaxT* at Guwahati is 37.0 Degree Celsius.

Also by the inequality (2.10),

36.9 Degree Celsius < *NAMaxT* at Guwahati < 37.1 Degree Celsius

which means, *NAMaxT* at Guwahati = 37.0 Degree Celsius.

2. Value of *NAMaxT* based on all distinct observed values excluding the two extreme observations namely the 1st & the 23rd ones:

Let us construct the following table:

Table – 3.1.4

Mean of all observed values excluding the corresponding one (in Degree Celsius)

Serial No	Observed value	Mean	Serial No	Observed value	Mean	Serial No	Observed value	Mean
1			9	36.5	37.0700	17	37.5	37.0200
2	35.3	37.1300	10	36.6	37.0650	18	38.0	36.9950
3	35.7	37.1100	11	36.7	37.0600	19	38.4	36.9750
4	35.8	37.1050	12	36.8	37.0550	20	38.6	36.9650
5	36.0	37.0950	13	37.1	37.0400	21	39.0	36.9450
6	36.1	37.0900	14	37.2	37.0350	22	39.2	36.9350
7	36.3	37.0800	15	37.3	37.0300	23		
8	36.4	37.0750	16	37.4	37.0250			

In this case, it is observed that negative error associated to estimate is decreasing from the last estimate '36.935' upwards and the positive error from the 1st estimate '37.13' downwards approaching point '37.0' where error is zero.

This means, the value of *NAMaxT* at Guwahati is 37.0 Degree Celsius.

Also by the inequality (2.15),

36.935 Degree Celsius < *NAMaxT* at Guwahati < 37.13 Degree Celsius

which means, *NAMaxT* at Guwahati = 37.0 Degree Celsius

3. Value of *NAMaxT* based on all distinct observed values excluding the four extreme observations namely the 1st, 2nd, 22nd & the 23rd ones:

Let us construct the following table:

Table – 3.1.5

Mean of all observed values excluding the corresponding one (in Degree Celsius)

Serial No	Observed value	Mean	Serial No	Observed value	Mean	Serial No	Observed value	Mean
1			9	36.5	37.05	17	37.5	36.9944
2			10	36.6	37.0444	18	38.0	36.9666
3	35.7	37.0944	11	36.7	37.03888	19	38.4	36.9444
4	35.8	37.0888	12	36.8	37.0333	20	38.6	36.9333
5	36.0	37.0777	13	37.1	37.0166	21	39.0	36.9111
6	36.1	37.0722	14	37.2	37.0111	22		
7	36.3	37.0611	15	37.3	37.0055	23		
8	36.4	37.0555	16	37.4	37.0			

In this case, it is observed that negative error associated to estimate is decreasing from the last estimate '36.9111' upwards and the positive error from the 1st estimate '37.0944' downwards approaching point '37.0' where error is zero.

This means, the value of *NAMaxT* at Guwahati is 37.0 Degree Celsius.

Also by the inequality (2.20),

$$36.911 \text{ Degree Celsius} < NAMaxT \text{ at Guwahati} < 37.09 \text{ Degree Celsius}$$

which means, *NAMaxT* at Guwahati = 37.0 Degree Celsius

3. Value of *NAMaxT* based on all distinct observed values excluding the six extreme observations namely the first three & the last three:

Let us construct the following table:

Table – 3.1.6

Mean of all observed values excluding the corresponding one (in Degree Celsius)

Serial No	Observed value	Mean	Serial No	Observed value	Mean	Serial No	Observed value	Mean
1			9	36.5	37.01250	17	37.5	36.95000
2			10	36.6	37.00625	18	38.0	36.91875
3			11	36.7	**37.0 ***	19	38.4	36.89375
4	35.8	37.05625	12	36.8	36.99375	20	38.6	36.88125
5	36.0	37.04375	13	37.1	36.97500	21		
6	36.1	37.0375	14	37.2	36.96875	22		
7	36.3	37.02500	15	37.3	36.9625	23		
8	36.4	37.01875	16	37.4	36.95625			

In this case, it is observed that negative error associated to estimate is decreasing from the last estimate '36.88125' upwards and the positive error from the 1^{st} estimate '37.05625' downwards approaching point '37.0' where error is zero.

This means, the value of *NAMaxT* at Guwahati is 37.0 Degree Celsius.

Also by the inequality (2.24),

36.88125 Degree Celsius < *NAMaxT* at Guwahati < 37.05625 Degree Celsius

Of course, this inequality does not imply that

NAMaxT at Guwahati = 37.0 Degree Celsius

Note: The true value of the *NAMaxT* at Guwahati can thus be confirmed to be 37.0 Degree Celsius.

3-2. Determination of the *NAMinT* at Guwahati

Observed values of Annual Minimum Temperature at Guwahati observed during the period from 1969 to 2010 have been collected from the meteorological department of India [6] & [7]. These have been presented in Table - IV and arranged in ascending order of magnitude in Table - V. Table - VI has been constructed for interval values of the Natural Annual Minimum Temperature **(abbreviated as *NAMinT*)** at Guwahati applying the inequalities (3.6), (3.9), (3.12) & (3.15).

Table – 3.2.1

Observed values of Annual Minimum Temperature at Guwahati (in Degree Celsius)

Year	Observed value	Year	Observed value	Year	Observed value	Year	Observed value
1969	5.8	1979	6.2	1989	6.7	2002	8.6
1970	7.2	1980	6.4	1990	8.7	2003	8.0
1971	5.9	1981	7.5	1991	7.4	2004	6.7
1972	8.0	1982	6.2	1992	5.9	2005	8.4
1973	5.0	1983	4.9	1993	7.8	2006	9.6
1974	6.3	1984	6.1	1994	8.8	2007	6.4
1975	7.2	1985	7.8	1995	7.5	2008	9.7
1976	6.6	1986	8.6	1996	9.4	2009	9.8
1977	6.2	1987	7.7	2000	8.5	2010	8.6
1978	7.3	1988	9.2	2001	8.9		

Table – 3.2.2

Observed values of Annual Minimum Temperature at Guwahati in ascending order (in Degree Celsius)

Serial No	Observed value	Serial No	Observed value	Serial No	Observed value	Serial No	Observed value
1	4.9	8	6.4	15	7.7	22	8.8
2	5.0	9	6.6	16	7.8	23	8.9
3	5.8	10	6.7	17	8.0	24	9.2
4	5.9	11	7.2	18	8.4	25	9.4
5	6.1	12	7.3	19	8.5	26	9.6
6	6.2	13	7.4	20	8.6	27	9.7
7	6.3	14	7.5	21	8.7	28	9.8

3.2.1. Computation of the Value of *NAMinT*:

Value of the *NAMinT* at Guwahati has been determined from all the distinct observations and also from all the distinct observations excluding some extreme observations as shown below:

1. Value of *NAMinT* based on all distinct observed values:

For determining the value of *NAMinT*, one requires to construct the following table:

Table – 3.2.3

Mean of all observed values excluding the corresponding one (in Degree Celsius)

Serial No	Observed value	Mean	Serial No	Observed value	Mean	Serial No	Observed value	Mean
1	4.9	7.68518	11	7.2	**7.60 ***	21	8.7	7.54444
2	5.0	7.68148	12	7.3	7.59629	22	8.8	7.54074
3	5.8	7.65185	13	7.4	7.59259	23	8.9	7.53703
4	5.9	7.64814	14	7.5	7.58888	24	9.2	7.52592
5	6.1	7.64074	15	7.7	7.58148	25	9.4	7.51851
6	6.2	7.63703	16	7.8	7.57777	26	9.6	7.51111
7	6.3	7.63333	17	8.0	7.57037	27	9.7	7,50740
8	6.4	7.62962	18	8.4	7.55555	28	9.8	7.50370
9	6.6	7.62222	19	8.5	7.55185	21	8.7	7.54444
10	6.7	7.61851	20	8.6	7.54814	22	8.8	7.54074

It is observed that negative error associated to estimate is decreasing from the last estimate '7.54074' upwards and the positive error from the 1st estimate '7.68518' downwards approaching point '7.6' where error is zero.

This means, the value of the *NAMinT* at Guwahati is 7.6 Degree Celsius.

Also by the inequality (2.10),

7.5037 Degree Celsius < *NAMaxT* at Guwahati < 7.68518 Degree Celsius

which means, *NAMinT* at Guwahati = 7.6 Degree Celsius

2. Value of *NAMinT* based on the distinct observed values excluding the two extreme ones namely the 1st & the 28th ones:

Let us construct the following table:

Table – 3.2.4

Mean of all observed values excluding the corresponding one (in Degree Celsius)

Serial No	Observed value	Mean	Serial No	Observed value	Mean	Serial No	Observed value	Mean
1			11	7.2	7.62000	21	8.7	7.56000
2	5.0	7.70800	12	7.3	7.61600	22	8.8	7.55600
3	5.8	7.67600	13	7.4	7.61200	23	8.9	7.55200
4	5.9	7.67200	14	7.5	7.60800	24	9.2	7.54000
5	6.1	7.66400	15	7.7	**7.60 ***	25	9.4	7.53200
6	6.2	7.66000	16	7.8	7.59600	26	9.6	7.52400
7	6.3	7.65600	17	8.0	7.58800	27	9.7	7,52000
8	6.4	7.65200	18	8.4	7.57200	28		
9	6.6	7.64400	19	8.5	7.56800			
10	6.7	7.64000	20	8.6	7.56400			

In this case, it is observed that negative error associated to estimate is decreasing from the last estimate ‘7.52’ upwards and the positive error from the 1st estimate ‘7.708’ downwards approaching point ‘7.6’ where error is zero.

This means, the value of the *NAMinT* at Guwahati is 7.6 Degree Celsius.

Also by the inequality (2.15),

7.52 Degree Celsius < *NAMaxT* at Guwahati < 7.70 Degree Celsius

which means, *NAMinT* at Guwahati = 7.6 Degree Celsius.

3. Value of *NAMinT* based on the distinct observed values excluding the four Extreme observations namely the 1st, 2nd, 27th & the 28th ones:

Let us construct the following table:

Table – 3.2.5

Mean of all observed values excluding the corresponding one (in Degree Celsius)

Serial No	Observed value	Mean	Serial No	Observed value	Mean	Serial No	Observed value	Mean
1			11	7.2	7.64347	21	8.7	7.57826
2			12	7.3	7.63913	22	8.8	7.57391
3	5.8	7.70434	13	7.4	7.63478	23	8.9	7.56956
4	5.9	7.70000	14	7.5	7.63043	24	9.2	7.55652
5	6.1	7.69130	15	7.7	7.62173	25	9.4	7.54782
6	6.2	7.68695	16	7.8	7.61739	26	9.6	7.53913
7	6.3	7.68260	17	8.0	7.60869	27		
8	6.4	7.67826	18	8.4	7.59130	28		
9	6.6	7.66956	19	8.5	7.58695			
10	6.7	7.66521	20	8.6	7.58260			

In this case, it is observed that negative error associated to estimate is decreasing from the last estimate '7.53913' upwards and the positive error from the 1st estimate '7.70434' downwards approaching point '7.6' where error is zero.

This means, the value of the *NAMinT* at Guwahati is 7.6 Degree Celsius.

Also by the inequality (2.20),

7.53913 Degree Celsius < *NAMinT* at Guwahati < 7.70434 Degree Celsius

which means, *NAMinT* at Guwahati = 7.6 Degree Celsius.

4. Value of *NAMinT* based on the distinct observed values excluding the six extreme observations namely the first three & the last three:

Let us construct the following table:

Table – 3.2.6

Mean of all observed values excluding the corresponding one (in Degree Celsius)

Serial No	Observed value	Mean	Serial No	Observed value	Mean	Serial No	Observed value	Mean
1			11	7.2	7.63809	21	8.7	7.56667
2			12	7.3	7.63333	22	8.8	7.56190
3			13	7.4	7.62857	23	8.9	7.55714
4	5.9	7.70000	14	7.5	7.62380	24	9.2	7.54285
5	6.1	7.69047	15	7.7	7.61428	25	9.4	7.53333
6	6.2	7.68571	16	7.8	7.60952	26		
7	6.3	7.68095	17	8.0	**7.60 ***	27		
8	6.4	7.67619	18	8.4	7.58095	28		
9	6.6	7.66667	19	8.5	7.57619			
10	6.7	7.66190	20	8.6	7.57142			

In this case, it is observed that negative error associated to estimate is decreasing from the last estimate '7.5333' upwards and the positive error from the 1st estimate '7.70' downwards approaching point '7.6' where error is zero.

This means, the value of the *NAMinT* at Guwahati is 7.6 Degree Celsius.

Also by the inequality (2.24),

7.5333 Degree Celsius < *NAMinT* at Guwahati < 7.70 Degree Celsius

which means, *NAMinT* at Guwahati = 7.6 Degree Celsius

Note: The true value of the *NAMinT* at Guwahati can thus be confirmed to be 7.6 Degree Celsius.

4. DISCUSSION

1. The method developed here can be summarized as follows:
   - (i) Arrange the distinct observed values in ascending or descending order of magnitude.
   - (ii) Corresponding to each observed value; compute the mean of the distinct observed all the distinct values excluding the former.
   - (iii) Observe the movements of the means from the highest as well as from the lowest ones and determine the value of the parameter

(iv) The value of the parameter can also be determined from the interval formed by the highest mean and the lowest mean.

(v) Confirm the correctness, of the result obtained, by repeating the process based on the observed values excluding

the extreme two observed values,
the extreme four observed values,
the extreme six observed values,
……………………………………

etc. respectively as required .

2. The existing statistical methods of estimation yield estimates which are not free from error. However, the method developed here yield the estimate which is free from error (i.e. exactly equal to the true value of the parameter).
3. It may be possible to apply this method in the determination of the true values of the parameters associated to the polynomial curves and to some other types of curves. However, it is yet to be investigated.
4. The estimated value computed by the existing methods of estimation varies if some observations are excluded and / or if some new observations are included. However, the value computed by the method developed here remains the same under this situation. Following findings (in **Table – 4.1, Table – 4.2, Table – 4.3 & Table – 4.4**) are some examples:

Table – 4.1

Maximum Likelihood / Minimum Variance Unbiased / Least Squares / Method of Moments / Minimum Chi Square Estimate of the *NAMaxT* at Guwahati if only one observation is excluded (in Degree Celsius)

Serial No	Excluded observation	Estimated value	Serial No	Excluded observation	Estimated value	Serial No	Excluded observation	Estimated value
1	37.1	37.0591	9	36.8	37.0727	17	36.3	37.0955
2	36.6	37.0818	10	38.6	36.9909	18	37.4	37.0455
3	36.0	37.1000	11	35.1	37.1500	19	39.4	36.9545
4	35.7	37.1200	12	35.8	37.1100	20	36.4	37.0909
5	39.0	36.9727	13	36.5	37.0864	21	37.5	37.0409
6	36.1	37.1045	14	36.7	37.0772	22	37.3	37.0500
7	39.2	36.9636	15	37.2	37.0545	23	38.0	37.0182
8	35.3	37.1400	16	38.4	37.0000			

However, in each of these situations, the value of the *NAMaxT* at Guwahati by the method developed here has been found to be 37.0 Degree Celsius.

Table – 4.2

Maximum Likelihood / Minimum Variance Unbiased / Least Squares / Method of Moments / Minimum Chi Square Estimate of the *NAMinT* at Guwahati if only one observation is excluded (in Degree Celsius)

Serial No	Excluded observation	Estimated value	Serial No	Excluded observation	Estimated value	Serial No	Excluded observation	Estimated value
1	5.8	7.65185	11	7.5	7.58888	21	8.8	7.54074
2	7.2	7.60000	12	4.9	7.68518	22	9.4	7.51851
3	5.9	7.64814	13	6.1	7.64074	23	8.5	7.55185
4	8.0	7.57037	14	7.8	7.57777	24	8.9	7.53703
5	5.0	7.68148	15	8.6	7.54814	25	8.4	7.55555
6	6.3	7.63333	16	7.7	7.58148	26	9.6	7.51111
7	6.6	7.62222	17	9.2	7.52592	27	9.7	7,50740
8	6.2	7.63703	18	6.7	7.61851	28	9.8	7.50370
9	7.3	7.59629	19	8.7	7.54444			
10	6.4	7.62962	20	7.4	7.59259			

However, in each of these situations, the value of the *NAMinT* at Guwahati by the method developed here has been found to be 7.6 Degree Celsius.

Table – 4.3

Maximum Likelihood / Minimum Variance Unbiased / Least Squares / Method of Moments / Minimum Chi Square Estimate of the *NAMaxT* at Guwahati if extreme observations are (in Degree Celsius)

Serial No	Excluded observation	Estimated value (in Degree Celsius)	Serial No	Excluded observation	Estimated value (in Degree Celsius)
1	None	37.06086	6	1st, 27th & 28th	36.93500
2	1st	37.15000	7	1st , 2nd, 27th & 28th	37.02105
3	28th	36.95450	8	1st , 2nd, 3rd, 27th & 28th	37.09440
4	1st & 28th	37.04280	9	1st , 2nd, 26th, 27th & 28th	36.91110
5	1st , 2nd & 28th	37.13000	10	1st , 2nd, 3rd, 26th, 27th & 28th	36.98230

However, in each of these situations, the value of the *NAMaxT* at Guwahati by the method developed here has been found to be 37.0 Degree Celsius.

Table – 4.4

Maximum Likelihood / Minimum Variance Unbiased / Least Squares / Method of Moments / Minimum Chi Square Estimate of the *NAMinT* at Guwahati if extreme observations are (in Degree Celsius)

Serial No	Excluded observation	Estimated value	Serial No	Excluded observation	Estimated value
1	None	7.58571	6	1^{st}, 27^{th} & 28^{th}	7.52000
2	1^{st}	7.68518	7	1^{st} , 2^{nd}, 27^{th} & 28^{th}	7.62500
3	28^{th}	7.50370	8	1^{st} , 2^{nd}, 3^{rd}, 27^{th} & 28^{th}	7.70434
4	1^{st} & 28^{th}	7.60384	9	1^{st} , 2^{nd}, 26^{th}, 27^{th} & 28^{th}	7.53913
5	1^{st} , 2^{nd} & 28^{th}	7.70800	10	1^{st} , 2^{nd}, 3^{rd}, 26^{th}, 27^{th} & 28^{th}	7.61816

However, in each of these situations, the value of the *NAMinT* at Guwahati by the method developed here has been found to be 7.6 Degree Celsius.

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