Tendency of Integral Valued Numerical Data: Method of Determination

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Abstract

One formulation/method has been developed for determining the tendency of integral valued numerical data. Derivation of the formulation has been discussed in this article. Moreover, numerical verification of the formulation has been shown by its application in data on rainfall (number of rainy days) in Mumbai.

Keywords: Integral valued numerical data, Tendency of data, formulation of determination

1. Introduction:

A basic statistical concept associated with the characteristic of data is the central tendency which means "the tendency of data to cluster around some central value [Chakrabarty (2015*a* , 2020*f* , 2021*f* , 2021*h* , 2021*j* , 2021*g*) ; Dodge (2003) , Upton and Cook (2008), Weisberg (1992)]. A measure of central tendency, which is based on average [Bakker (2003) ; Chakrabarty (2015*b*) ; Dodge (2003) ; Miguel (2016) ; Upton and Cook (2008)], is a typical value that lies in the central /middle part of the data The existing measures of central tendency [Chakrabarty (2022*c*) ; Dodge (2003) ; Manikandan (2011) ; Upton and Cook (2008) ; Williams (1984)], which is based on measure of average [Chakrabarty (2017*a* , 2018*a* , 2020*a*) ; Dodge (2003) ; Kolmogorov (1930) ; Miguel(2016); Upton and Cook (2008)], can be placed in two broad divisions namely mathematical measures and positional measures. The three popular mathematical measures of central tendency of data are Arithmetic Mean (AM), Geometric Mean (GM), and Harmonic Mean (HM), also known as Pythagorean Means [Chakrabarty (2016*a* , [Chakrabarty (2019*a* , 2019*g* , 2021*b* , 2021*e* , 2021*h*)] introduced by Pythagoras [Christoph (2005)] which had later been generalized in order to obtain more general definitions of average [Chakrabarty (2018*b* , 2018*c* , 2018*d* , 2018*e* , 2018*h* , 2019*b* , 2019*c* , 2019*d* , 2021*f*)]. The positional measures of central tendency are Median and Mode [Manikandan (2011) ; Williams (1984)] .

Some more mathematical measures of central tendency namely Arithmetic-Geometric Mean (AGM) [Chakrabarty (2020*b* , 2021*a* , 2021*e* , 2021*i* , 2022*a*)], Arithmetic-Harmonic Mean (AHM) [David (1984) ; Chakrabarty (2020*c* , 2020*d* , 2021*a* , 2021*c* , 2021*d* , 2021*e* , 2021*i* , 2022*a* ; Hazewinkel (2001) ; Foster and Phillips (1984)], Geometric-Harmonic Mean (GHM) [Chakrabarty (2020*e* , 2021*a* , 2021*e* , 2021*i* , 2021*j* , 2022*a*)], Arithmetic-Geometric-Harmonic Mean (AGHM) [Chakrabarty (2020*f* , 2020*g* , 2021*a* , 2021*e* , 2021*i* , 2022*a*)] and GM of AM & HM [Chakrabarty (2022*b*)] have been developed in some recent studies. Moreover, one method has recently been developed for measuring central tendency of integral valued numerical data [Chakrabarty (2022*c*)].

Each measure of central tendency results in value which lies in the middle part or central part of the associated data. However, tendency of data may not always be towards the central portion of the data in reality. There are situations in reality where the tendency of data is not towards the central / middle portion of the data but towards one end point of the data set. As an example, suppose that a region/location normally remains completely drought (i.e. without rainfall i.e. with zero rainy day) during a particular time period. But, due to some irregular/accidental/random factor/cause there may be rainfall (though rare of very small) during that time period [Basak and Sahu (2019) ; Gharphalia. et al (2018) ; Chakrabarty (2014 , 2019f , 2021k) ; Hills (1974) ; Jose et al (2020) ; Kumar, Jain and Singh (2010) ; Krishnakumar et al (2009) ; Nikumbh, Chakraborty and Bhat (2019)]. As a result, if data on number of rainy days are collected and tendency is calculated by the existing measures then the value obtained will be different from 0 and > 0. But, the real/actual tendency of number of rainy days in this case is 0. Moreover, there are situation(s) where data set consists of integral valued numbers so that the tendency of the data set is also an integral valued number (for example, number of rainy days as mentioned above). In such situation(s), the above mathematical measures may fail to provide the value, which is a valid one, of the tendency of data since the values provided by them are not bound to be integers. The mathematical measures may suit the continuous data. Positional measures may suit ordinal data. Thus there is necessity of some formulation/method of determining tendency of data in such situation. For this reason, one formulation/method has been developed for determining the tendency of integral valued numerical data. This formulation/method has been discussed in this article. Moreover, attempt has been made on numerical verification of the formulation/method by its application in data on rainfall (number of rainy days) at Mumbai. This paper describes the derivation of the formulation/method and its numerical verification with the help of data on number of rainy days at Mumbai.

2. Integral Valued Data – Method of Determination of Tendency:

Let us first consider non-negative integral valued data.

Suppose,

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

are N observations which are non-negative integral values, observed on a non-negative integral valued random variable X , so that its central tendency is also a non-negative integral value.

Arithmetic Mean (AM) of the N observations is defined by

$$AM\,(x_1 , x_2 , \ldots\ldots\ldots , x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i \qquad (1)$$

[Christoph (2005) ; Chakrabarty (2016*a* , 2018*a*)].

If μ is the central tendency of X, the observations can be expressed as

$$x_i = \mu + \varepsilon_i \;, \quad (i = 1 , 2 , \ldots\ldots\ldots , N) \qquad (2)$$

where

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_N$$

are the respective errors associated to the observations which are random and assume integral values.

This implies,

$$AM\,(x_1\,, x_2\,,\ \ldots\ldots\ldots\ldots, x_N) = \mu + AM\,(\varepsilon_1\,, \varepsilon_2\,,\ \ldots\ldots\ldots\ldots, \varepsilon_N) \qquad (3)$$

where

$$AM\,(\varepsilon_1\,, \varepsilon_2\,,\ \ldots\ldots\ldots\ldots, \varepsilon_N) = \frac{1}{N}\ \sum_{i=1}^{N} \varepsilon_i \qquad (4)$$

When the central tendency is 0 i.e. $\mu = 0$,

then the only possibility is that some of

$$\varepsilon_1\,, \varepsilon_2\,,\ \ldots\ldots\ldots\ldots, \varepsilon_N$$

may be 0 while others are strictly positive integers (since the observations are non-negative integral values) so that due to their randomness $AM\,(\varepsilon_1\,, \varepsilon_2\,,\ \ldots\ldots\ldots\ldots, \varepsilon_N)$ is a positive real number and it becomes smaller and smaller as N becomes larger and larger i.e.

$$AM\,(\varepsilon_1\,, \varepsilon_2\,,\ \ldots\ldots\ldots\ldots, \varepsilon_N) \rightarrow 0 \quad \text{as} \quad N \rightarrow \propto \qquad (5)$$

[Sen and Singer (1993) ; Seneta, (2013)].

Hence for large N, $AM\,(\varepsilon_1\,, \varepsilon_2\,,\ \ldots\ldots\ldots\ldots, \varepsilon_N)$ will be very near to 0 but > 0.

Therefore, the integer just below $AM\,(x_1\,, x_2\,,\ \ldots\ldots\ldots\ldots, x_N)$ is to be the value of central tendency of *X*.

When the central tendency is 1 i.e. $\mu = 1$,

then the possibility is that some of

$$\varepsilon_1\,, \varepsilon_2\,,\ \ldots\ldots\ldots\ldots, \varepsilon_N$$

may be -1 , some may be 0 while others are strictly positive integers so that due to their randomness $AM\,(\varepsilon_1\,, \varepsilon_2\,,\ \ldots\ldots\ldots\ldots, \varepsilon_N)$ is a positive real number and it becomes smaller and smaller as N becomes larger and larger.

By the same logic as in the earlier case, the integer just below $AM\,(x_1\,, x_2\,,\ \ldots\ldots\ldots\ldots, x_N)$ will be the value of central tendency of *x*.

When the central tendency is m i.e. $\mu = m$,

then the possible values of errors are

$$-m,\ -m+1\ ,-m+2\ ,\ \ldots\ldots\ldots\ ,-3\ ,-2\ ,-1\ ,0\ ,1\ ,2\ ,3\ ,\ldots\ldots\ldots\ ,\ m,\ m+1\ ,\ m+2\ ,\ldots\ldots\ldots$$

Due to their randomness, the theoretical/expected value of $AM\ (\varepsilon_1\ ,\varepsilon_2\ ,\ \ldots\ldots\ldots\ ,\varepsilon_N)$ is a positive real number for data size greater than $2m$ and it becomes smaller and smaller as N becomes larger and larger.

Hence by the same logic as earlier, the integer just below $AM\ (x_1\ ,x_2\ ,\ \ldots\ldots\ldots\ ,x_N)$ is the theoretical/expected value of central tendency of x.

But for finite (though large) set of data the value of $AM\ (\varepsilon_1\ ,\varepsilon_2\ ,\ \ldots\ldots\ldots\ ,\varepsilon_N)$ may not always be a positive real number. It may also be negative. But whether positive or negative, it becomes closer and closer to 0 as N becomes larger and larger i.e.

$$AM\ (\varepsilon_1\ ,\varepsilon_2\ ,\ \ldots\ldots\ldots\ ,\varepsilon_N) \to 0 \quad \text{as } N \to \propto \tag{6}$$

[58 , 59].

Hence for large N, $AM\ (x_1\ ,x_2\ ,\ \ldots\ldots\ldots\ ,x_N)$ will be very near to 0

Therefore,

either the integer just below $AM\ (x_1\ ,x_2\ ,\ \ldots\ldots\ldots\ ,x_N)$

or the integer just above $AM\ (x_1\ ,x_2\ ,\ \ldots\ldots\ldots\ ,x_N)$

is the value of central tendency of X in this case.

Again, since the tendency of the data set is μ, it is most likely to occur maximum times in the data set and certain to occur maximum times in the whole population.

This implies that mode is theoretically the central tendency of the data set which further implies that mode tends to be identical with the value of central tendency as the size of data set tends to be large.

Thus for finite (however large) set of data, the value of

the integer just below $AM\ (x_1\ ,x_2\ ,\ \ldots\ldots\ldots\ ,x_N)$

or the integer just above *AM* **(**x_1 , x_2 , ………… , x_N**)**

which is identical with the *M*ode (x_1 , x_2 , ………… , x_N) is the value of central tendency in this case.

Of course, for finite (though large) set of data it may not be so.

Possible Situations and determination of Central Tendency

(1) If either of the integer just below *AM* **(**x_1 , x_2 , ………… , x_N**)** or the integer just above *AM* **(**x_1 , x_2 , ………… , x_N**)** is identical with the *M*ode (x_1 , x_2 , ………… , x_N) of the data set then that common value is the value of central tendency of *x*.

(2) If none of the integer just below *AM* **(**x_1 , x_2 , ………… , x_N**)** and the integer just above *AM* **(**x_1 , x_2 , ………… , x_N**)** is identical with the mode of the data set then identify the outlier(s) in the data set and repeat the process to obtain the common value of *M*ode (x_1 , x_2 , ………… , x_N) and either of the integer just below *AM* (x_1 , x_2 , ………… , x_N) or the integer just above *AM* **(**x_1 , x_2 , ………… , x_N**)** which is the value of central tendency of *x*.

(3) If *M*ode (x_1 , x_2 , ………… , x_N) is found not to be unique and/or if found not to be identifiable, then the integer just below *AM* **(**x_1 , x_2 , ………… , x_N**)** or the integer just above *AM* **(**x_1 , x_2 , ………… , x_N**)** is the value of central tendency of *X* depending upon the set of possible positive errors associated to x_1 , x_2 , ………… , x_N is bigger than the set of possible negative errors associated to them or the set of possible positive errors is smaller than the set of possible negative errors.

Note:

(1) The above formulation/method is for non-negative integral valued observations.

(2) If the observations are all non-positive integral numbers then by changing the origin, the observations can be transformed to all non-negative integral numbers so that the method can be applied to determine the tendency of the transformed data.

If c is an arbitrary number which transforms all the original observations to non-negative integral numbers then

$$\text{Tendency of original data} = \text{Tendency of transformed data} - c \tag{7}$$

(3) If the observations are mixture of non-negative and non-positive integral valued then the technique mentioned in (2) can be applied.

3. Numerical Example:

Data on number of rainy days (month-wise) at Mumbai occurred in each of the twelve months during the period from 1969 to 2001 have been shown in **Table - 1**.

Table - 1
(Number of Rainy Days at Mumbai)

Year	Number of Rainy Days in the month											
	Jan	Feb	Mar	April	May	June	Jul	Aug	Sept	Oct	Nov	Dec
1969	0	0	0	0	0	14	28	20	15	1	2	0
1970	0	0	0	1	1	18	19	27	17	5	0	0
1971	0	0	0	0	2	17	19	18	11	2	0	0
1972	0	1	0	0	0	0	22	12	6	0	1	0
1973	0	0	0	0	0	12	26	26	25	2	0	0
1974	0	0	0	1	3	10	24	26	18	10	0	0
1975	0	0	0	0	0	15	20	27	16	9	0	0
1976	0	0	0	0	0	15	24	22	16	0	2	0
1977	0	0	0	0	0	13	27	16	11	3	5	0
1978	0	0	0	0	0	19	24	23	13	2	3	1
1979	0	0	0	0	0	12	17	17	10	1	7	0
1980	0	0	0	0	0	15	17	25	10	2	2	2
1981	0	0	0	0	0	0	24	21	18	6	1	0
1982	0	0	0	0	0	0	20	24	15	0	2	0
1983	0	0	0	1	0	12	27	25	23	5	0	0
1984	0	1	0	0	0	14	24	19	9	3	0	0

1985	0	0	0	0	1	18	22	20	8	6	0	0
1986	0	0	0	0	0	16	16	15	6	0	2	1
1987	0	0	0	0	0	14	26	21	5	3	0	2
1988	0	0	0	0	0	17	27	24	25	3	0	0
1989	0	0	0	0	0	18	24	23	13	3	0	0
1990	0	1	1	0	5	13	23	27	21	6	0	0
1991	1	0	0	0	0	10	26	26	6	0	0	1
1992	0	0	0	0	0	9	22	22	11	3	0	0
1993	0	0	0	0	0	9	26	23	22	7	0	0
1994	1	0	0	0	1	13	28	21	16	2	0	0
1995	0	0	0	0	0	5	24	17	13	5	0	0
1996	0	0	0	0	0	12	29	25	15	6	0	0
1997	0	0	0	0	0	16	16	24	12	0	4	2
1998	0	0	0	0	0	16	22	19	14	12	2	0
1999	0	0	0	0	4	18	18	16	16	6	0	0
2000	0	0	0	0	7	4	19	19	7	4	0	1

(Source: Indian Meteorological Department, Pune)

Tendency of Number of Rainy Days in January

Cumulative arithmetic mean (CAM) and cumulative mode (CM) of number of rainy days have been calculated from observations in this data set and have been listed in **Table - 2**.

Table - 2

(Values of CAM & CM of Number of Rainy in January)

Year	CAM	CM	Year	CAM	CM	Year	CAM	CM
1969	0		1980	0	0	1991	0.043	0
1970	0		1981	0	0	1992	0.042	0
1971	0		1982	0	0	1993	0.04	0
1972	0	0	1983	0	0	1994	0.077	0
1973	0	0	1984	0	0	1995	0.074	0
1974	0	0	1985	0	0	1996	0.071	0
1975	0	0	1986	0	0	1997	0.069	0
1976	0	0	1987	0	0	1998	0.067	0
1977	0	0	1988	0	0	1999	0.065	0
1978	0	0	1989	0	0	2000	0.063	0
1979	0	0	1990	0	0			

In **Table - 2**, it is observed that for large number (> 30) of observations, integral value just less than arithmetic mean is 0 & mode is also 0.

Thus, the common value of mode and integral value just less than arithmetic mean for large number (> 30) of observations is 0.

Hence, the tendency of number of rainy days at Mumbai in the month of January is 0.

Similarly, the common value of mode and integral value just less than arithmetic mean for large number (> 30) of observations in each of the other months except September and November has been found as follows (shown in **Table - 3**) which are the values of tendencies of rainy days in the corresponding months:

Table - 3

(Common value of CM and Integral value just less than CAM)

Month	Value	Month	Value	Month	Value
February	0	May	0	August	21
March	0	June	12	October	3
April	0	July	22	December	0

Tendency of Number of Rainy Days in September

Values of CAM and CM of number of rainy days in this month have been calculated from observations in this data set and have been listed in **Table - 4**.

Table - 4

(Values of CAM & CM of Number of Rainy in September)

Year	CAM	CM	Year	CAM	CM	Year	CAM	CM
1969	15		1980	14		1991		
1970	16		1981	14.308		1992	13.667	
1971	14.333		1982	14.3578		1993	14	
1972	12.25		1983	14.9338		1994	14.077	
1973	14.8		1984	14.563		1995	14.0377	
1974	15.333		1985	14.176		1996	14.0717	
1975	15.429		1986	13.722		1997	14	
1976	15.5		1987	13.263		1998	14	

1977	13.778		1988	13.85		1999	14.065	16
1978	14.8		1989	13.809		2000	13.844	16
1979	14.364		1990	14.136				

In **Table - 4**, it is observed that for large number (> 30) observations, integral value just less than arithmetic mean is 13 while mode is 16 i.e. they are not identical. This may happen due to the presence of outlier(s) in the data. One outlier is 14.3 that occurs corresponding to the year 1973. Values of CAM and CM of number of rainy days calculated, excluding the outlier in this data set, have been shown in **Table - 5**.

Table - 5

(Values of CAM & CM of Number of Rainy in September calculated excluding outlier)

Year	CAM	CM	Year	CAM	CM	Year	CAM	CM
1969	15		1980	13		1991	12.714	
1970	16		1981	13.417		1992	12.636	
1971	14.333		1982	13.538		1993	13.043	
1972	12.25		1983	14.214		1994	13.167	
1973	XXXXXX		1984	13.867		1995	13.16	
1974	13.4		1985	13.5		1996	13.231	
1975	13.833		1986	13.059		1997	13.185	
1976	14.143		1987	12.611		1998	13.214	
1977	13.75		1988			1999	13.310	16
1978	13.667		1989	12.632		2000	13.1	16
1979	13.3		1990	13.05				

In **Table - 5** also, it is observed that the integral value just less than arithmetic mean is 13 while mode is 16 i.e. they are not identical.

However, it is observed that the value "16" of mode has appeared 4 times while the observation"13" has appeared 3 times.

Thus, the observation "13" may also be the mode if more observations are taken in the study.

Accordingly, the central tendency of number of rainy days in September is 0.

Hence, the tendency of number of rainy days at Mumbai in the month of September is 21.

Tendency of Number of Rainy Days in November

Values of CAM and CM of the number of rainy days in this month have been calculated from observations in this data set and have been listed in **Table - 6**.

Table - 6

(Values of CAM & CM of Number of Rainy in November)

Year	CAM	CM	Year	CAM	CM	Year	CAM	CM
1969	2		1980	1.833	0	1991	1.174	0
1970	1		1981	1.769	0	1992	1.125	0
1971	0.667		1982	1.786	0	1993	1.08	0
1972	0.75	0	1983	1.667	0	1994	1.038	0
1973	0.6	0	1984	1.563	0	1995	1	0
1974	0.5	0	1985	1.471	0	1996	0.964	0
1975	0.429	0	1986	1.5	0	1997	1.069	0
1976	0.625	0	1987	1.421	0	1998	1.1	0
1977	1.1119	0	1988	1.35	0	1999	1.065	0
1978	1.3	0	1989	1.286	0	2000	1.031	0
1979	1.818	0	1990	1.227	0			

In **Table - 6,** it is observed that for large number (> 30) observations, integral value just less than arithmetic mean is 1 while mode is 0 i.e. they are not identical.

This may happen due to the presence of outlier(s) in the data. One outlier is 7 that occurs corresponding to the year 1979.

CAM and CM of number of rainy days calculated, excluding the outlier in this data set, have been listed in **Table - 7**.

Table - 7
(Values of CAM & CM of Number of Rainy in November calculated excluding outlier)

Year	CAM	CM	Year	CAM	CM	Year	CAM	CM
1969	2		1980	1.364	0	1991	0.909	0
1970	1		1981	1.333	0	1992	0.869	0
1971	0.667		1982	1.385	0	1993	0.833	0
1972	0.75		1983	1.286	0	1994	0.8	0
1973	0.6	0	1984	1.2	0	1995	0.769	0
1974	0.5	0	1985	1.125	0	1996	0.741	0
1975	0.429	0	1986	1.176	0	1997	0.857	0
1976	0.625	0	1987	1.111	0	1998	0.897	0
1977	1.1119	0	1988	1.053	0	1999	0.867	0
1978	1.3	0	1989	1	0	2000	0.839	0
1979	XXXXXX		1990	0.952	0			

In **Table - 7**, it is observed that for large number (> 30) observations, integral value just less than the arithmetic mean is 0 & mode is also 0.

Thus, the common value of mode and integral value just less than arithmetic mean exists and is 0.

Hence, the tendency of number of rainy days at Mumbai in the month of November is 0.

4. Conclusion:

The formulation of tendency of non-negative integral valued data can be summarized as follows:

"The value of tendency of a data set containing non-negative integral valued numbers is the common value of the mode and the integer just below the arithmetic mean of the data set if exists and simply the integer just below the arithmetic mean if does not exist."

It is to be noted that when the common value does not exist, the presence of outlier(s) in the data set is to be examined and necessary rectification of calculation is to be worked out if the presence of outlier is found in the data set.

If the observations are all non-positive integral numbers then by changing the origin, the observations can be transformed to all non-negative integral numbers so that the method can be applied to determine the tendency of the transformed data and then the tendency of original data can be obtained by applying the formula (7). Similarly, if the observations are mixture of non-negative and non-positive integral numbers then also the same technique can be applied to obtain the tendency of the observations.

Each measure of central tendency, developed and used so far, results in value which lies in the middle part or central part of the associated data which implies that the tendency of data is towards the central portion of the data. However, tendency of data may not always be towards the central portion of the data in reality. It may be towards one end point of the data set. Thus, tendency can be regarded as a basic characteristic of data so that central tendency is a special case of this characteristic. It can be more appropriate to say "measure of tendency of data" than to say "measure of central tendency of data".

In the current study, attempt has been made on searching for measure of tendency of integral valued numerical data. It is yet to search for measure of tendency of data of other than integral type.

Finally, the concept of tendency of data can be regarded as a generalization of the concept of the central tendency of data. Accordingly, as per the definition of research [Chakrabarty (2018*f* , 2018*g* , 2019*e*)], it can be concluded that this work can be regarded as an extension type of research carrying fundamental importance in the theory of analysis of data.

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