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Sex Ratio in India and Seven Measures of Central Tendency

Dhritikesh Chakrabarty

Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India.
Email:

Abstract

An attempt has been made on establishing the four measures of average namely *AGM*, *AHM*, *GHM* and *AGHM* which had been derived from the three Pythagorean means namely *AM*, *GM* and *HM* as measures of central tendency of numerical data. These four measures of central tendency have been applied in estimating the central tendency of data on the sex ratio in the sates in India. This paper is based on the logical justification of the seven measures namely *AM*, *GM*, *HM*, *AGM*, *AHM*, *GHM* & *AGHM* of being measures of central tendency of numerical data and on the estimation of central tendency of sex ratio the sex ratio in the sates in India by the applications of these measures.

Keywords: Sex Ratio, Central Tendency, *AGM*, *AHM*, *GHM*, *AGHM.*

1. Introduction

Average [Baker (2003)] is a vital concept and averaging is a vital technique involved in most of the measures associated to data (or list of numerical values). It is Pythagoras [Riedweg, 2005], who defined the three most common measures of average namely Arithmetic Mean (*AM*), Geometric Mean (*GM*) and Harmonic Mean (*HM*) which were given the name "Pythagorean Means" [Chakrabarty (2016b , 2019f , 2021f , 2021g) , David (2004) , Kolmogorov (1933) , Miguel (2016)] as a mark of honor to him. A number of measures of average were developed in successive studies among which some are Quadratic Mean , Square Root Mean , Cubic Mean , Cube Root Mean , Generalized *p* Mean & Generalized p^{th} Root Mean etc [Chakrabarty (2016*a* , 2018*h* , 2019*f* , 2020*a*) , Kolmogorov (1930) , Miguel (2016)]. The next development was on generalized definitions of measure of average like Generalized *f* - Mean [Chakrabarty (2018*b* , 2018*c*)], Generalized f_H – Mean [Chakrabarty (2018*d*)] and Generalized f_G – Mean [Chakrabarty (2018*e* , 2019*b*)] and general method of defining average [Chakrabarty (2019*d* , 2019*e*)].

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In statistics, the three Pythagorean means [Chakrabarty (2021*b*)] are used in measuring the central tendency of numerical data [Plackett (1958) , Weisberg (1972) , Williams (1984)]. However, the accuracy of the value of central tendency yielded by each of the three Pythagorean means is not known. Recently, there have been a lot of studies on analysis of numerical data based on average in general and on Pythagorean means specially [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*a* , 2015*b* , 2015*c* , 2015*d* , 2015*e* , 2015*f* , 2015*g* , 2016*a* , 2017*b* , 2017*c* , 2017*d* , 2018*a* , 2019*a*)]. In the mean time, several attempts have been made on determining accurate value of central tendency of numerical data. However, still there is necessity of more accurate measure of the same. With an objective of finding out more accurate measure of central tendency of data, four measures of average namely Arithmetic-Geometric Mean (*AGM*) [David (2004) , Chakrabarty (2020*b* , 2021*a* , 2021*c* , 2021*d*) , Hazewinkel (2001) , Kolmogorov (1933)], Arithmetic-Harmonic Mean (*AHM*) [Chakrabarty (2020*c* , 2020*d* , 2021*a* , Foster & Phillips (1984)], Geometric-Harmonic Mean (*GHM*) [Chakrabarty (2020*e* , 2021*a*)] and Arithmetic-Geometric-Harmonic (*AGHM*) [Chakrabarty (2020*f* , 2021*a* , 2021*e* , 2021*f* , 2021*g* , 2021*h* , 2021*i* , 2021*j*)] have been derived from the three Pythagorean means. In the current study, an attempt has been made on establishing the four measures of average namely *AGM*, *AHM*, *GHM* and *AGHM* as measures of central tendency of numerical data. These four measures of central tendency have been applied in estimating the central tendency of data on the sex ratio in the sates in India. This paper is based on the logical justification of the seven measures namely *AM*, *GM*, *HM*, *AGM*, *AHM*, *GHM* & *AGHM* of being measures of central tendency of numerical data and on the estimation of central tendency of sex ratio the sex ratio in the sates in India by the applications of these measures.

2. Seven Measures of Average:

The seven measures of average considered here are *AM*, GM, *HM*, *AGM*, *AHM*, *GHM* and *AGHM*.

Let

$$x_1 , x_2 , \ldots\ldots\ldots\ldots , x_N$$

be *N* positive numbers or values or observations (not all equal or identical) which are strictly positive. The three measures of average namely *AM*, GM & *HM* of the observed values are defined as by

$$AM\,(x_1 , x_2 , \ldots\ldots , x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i , \tag{2.1}$$

$$GM\,(x_1 , x_2 , \ldots\ldots , x_N) = \left(\prod_{i=1}^{N} x_i \right)^{1/N} \tag{2.2}$$

& $HM(x_1\ ,\ x_2\ , \ldots\ldots\ldots\ , x_N) = (\frac{1}{N}\sum_{i=1}^{N} x_i^{\,-1})^{-1}$ **(2.3)**

Respectively which satisfy the inequality

Largest $(x_1\ , x_2\ , \ldots\ , x_N) > a_0 > g_0 > h_0 >$ Smallest$(x_1\ , x_2\ , \ldots\ , x_N)$ (2.4)

[Chakrabarty (2016b , 2019f , 2021f , 2021g) , David (2004)].

Now let

$a_0 = AM\,(x_1\ ,\ x_2\ , \ldots\ldots\ , x_N)$,

$g_0 = GM\,(x_1\ ,\ x_2\ , \ldots\ldots\ , x_N)$

& $h_0 = HM\,(x_1\ ,\ x_2\ , \ldots\ldots\ldots\ , x_N)$

Then *AGM*, *AHM*, *GHM* and *AGHM* of the observed values are defined as follows:

AGM:

The AGM of $x_1\ ,\ x_2\ , \ldots\ldots\ldots\ , x_N$, denoted by $AGM\ (x_1\ ,\ x_2\ , \ldots\ldots\ldots\ , x_N)$ is the common point of convergence M_{AG} of the two sequences $\{a_n\}$ & $\{g_n\}$ defined by

$$a_{n+1} = ½\,(a_n + g_n) \quad \& \quad g_{n+1} = (a_n \cdot g_n)^{1/2}$$

Respectively [David (2004) , Chakrabarty (2020*b* , 2021*a* , 2021*c* , 2021*d*) , Hazewinkel (2001)].

AHM:

The AHM of $x_1\ ,\ x_2\ , \ldots\ldots\ldots\ , x_N$, denoted by $AHM\ (x_1\ ,\ x_2\ , \ldots\ldots\ldots\ , x_N)$ is the common point of convergence M_{AH} of two sequences $\{a'_n\}$ & $\{h'_n\}$ defined by

$$a'_{n+1} = ½(a'_n + h'_n) \quad \& \quad h'_{n+1} = \{½(a'^{\,-1}_n + h'^{\,-1}_n)\}^{-1}$$

respectively where $a'_0 = a_0$ & $h'_0 = h_0$ [Chakrabarty (2020*c* , 2020*d* , 2021*a* , Foster & Phillips (1984)].

GHM:

The GHM of $x_1\ ,\ x_2\ , \ldots\ldots\ldots\ , x_N$, denoted by $GHM\ (x_1\ ,\ x_2\ , \ldots\ldots\ldots\ , x_N)$ is the common point of convergence M_{GH} of two the two sequences $\{g''_n\}$ & $\{h''_n\}$ defined by

$$g''_{n+1} = (g''_n\ .\ h''_n)^{1/2} \quad \& \quad h''_{n+1} = \{½(g''^{\,-1}_n + h''^{\,-1}_n)\}^{-1}$$

respectively where $g^{//}_0 = g_0$ & $h^{//}_0 = \boldsymbol{h}_0$ [Chakrabarty (2020*e* , 2021*a*)].

AGHM

The $AGHM$ of x_1 , x_2 , ……….. , x_N , denoted by $AGHM$ $(x_1$, x_2 , ……….. , $x_N)$ is the common point of convergence M_{AGH} of the three sequences $\{\mathbf{a}^{///}_n\}$, $\{\mathbf{g}^{///}_n\}$ & $\{\mathbf{h}^{///}_n\}$ defined respectively by

$$\mathbf{a}^{///}_{n+1} = 1/3\,(\mathbf{a}^{///}_n + \mathbf{g}^{///}_n + h^{///}_n)\ ,$$

$$\mathbf{g}^{///}_{n+1} = (\mathbf{a}^{///}_n\,\mathbf{g}^{///}_n\,\mathbf{h}^{///}_n)^{1/3}$$

$$\&\ \ \mathbf{h}^{///}_{n+1} = \{1/3\,(\mathbf{a}^{///\,-1}_n + \mathbf{g}^{///\,-1}_n + h^{///\,-1}_n)\}^{-1}$$

where $a^{///}_0 = \boldsymbol{a}_0$, $g^{///}_0 = g_0$ & $\boldsymbol{h}^{///}_0 = \boldsymbol{h}_0$ [Chakrabarty (2020*f* , 2021*a* , 2021*e* , 2021*f* , 2021*g* , 2021*h* , 2021*i* , 2021*j*)].

Note

When the observed values

$$x_1\ ,\ x_2\ ,\ \ldots\ldots\ldots\ldots\ ,\ x_N$$

are on sex ratio then the values are strictly positive real numbers. Hence in this case, all the three Pythagorean means namely *AM, GM* & *HM* exist and hence each of *AGM, AHM, GHM* & *AGHM* also exists.

3. Seven Measures of Central Tendency:

It will now be shown that each of *AGM, AHM, GHM* & *AGHM* can be a measure of central tendency of numerical data in addition to *AM, GM* & *HM.*

AM as a Measure of Central Tendency

If μ is the central tendency of the observations

$$x_1\ ,\ x_2\ ,\ \ldots\ldots\ldots\ldots\ ,\ x_N$$

in a set of quantitative data then the observations are composed of μ and random errors.

In other words, these can be described by or expressed as

$$x_i = \mu + \varepsilon_i\ \ ,\ \ (i = 1\ ,2\ ,\ \ldots\ldots\ldots\ ,N) \qquad (3.1)$$

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where $\varepsilon_1, \varepsilon_2, \ldots\ldots\ldots, \varepsilon_N$ are the random errors, which assume positive and negative values in random order, associated to $x_1, x_2, \ldots\ldots\ldots, x_N$ respectively.

In this case,

$$AM(x_1, x_2, \ldots\ldots\ldots, x_N) \to \mu \quad \text{as} \quad N \to \infty$$

where $$AM(x_1, x_2, \ldots\ldots\ldots, x_N) = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Accordingly, $AM(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be regarded as a measure of μ and consequently a measure of central tendency of $x_1, x_2, \ldots\ldots\ldots, x_N$.

GM as a Measure of Central Tendency

Again since the observations

$$x_1, x_2, \ldots\ldots\ldots, x_N$$

consist of μ and random errors,

these can be described by or expressed as

$$x_i = \mu\, \varepsilon_i^{//} \quad , \quad (i = 1, 2, \ldots\ldots\ldots, N) \tag{3.2}$$

where $\varepsilon_1^{//}, \varepsilon_2^{//}, \ldots\ldots\ldots, \varepsilon_N^{//}$ are the random errors, which assume values in (0 , 1) and in (1 , ∝) in random order, associated to $x_1, x_2, \ldots\ldots\ldots, x_N$ respectively.

In this case,

$$GM(x_1, x_2, \ldots\ldots\ldots, x_N) \to \mu \quad \text{as} \quad N \to \infty$$

where $$GM(x_1, x_2, \ldots\ldots\ldots, x_N) = \left(\prod_{i=1}^{N} x_i\right)^{1/N}$$

Accordingly, $GM(x_1, x_2, \ldots\ldots\ldots, x_N)$ can also be regarded as a measure of μ and consequently a measure of central tendency of $x_1, x_2, \ldots\ldots\ldots, x_N$.

HM as a Measure of Central Tendency

Again since the observations

$$x_1, x_2, \ldots\ldots\ldots, x_N$$

consist of μ and random errors,

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therefore, the reciprocals

$$x_1^{-1}, x_2^{-1}, \ldots\ldots\ldots, x_N^{-1}$$

are composed of μ^{-1} and random errors different from the respective random errors

$$\varepsilon_1, \varepsilon_2, \ldots\ldots\ldots, \varepsilon_N$$

provided $x_1, x_2, \ldots\ldots\ldots, x_N$ are all different from zero.

In this case thus

$$x_i^{-1} = \mu^{-1} + \varepsilon_i' \quad , \quad (i = 1, 2, \ldots\ldots\ldots, N) \qquad (3.3)$$

where $\varepsilon_1', \varepsilon_2', \ldots\ldots, \varepsilon_N'$ are the random errors, which assume positive and negative values in random order, associated to are the random errors associated to $x_1^{-1}, x_2^{-1}, \ldots\ldots\ldots, x_N^{-1}$ respectively.

In this case,

$$HM(x_1, x_2, \ldots\ldots\ldots, x_N) \longrightarrow \mu \text{ as } N \longrightarrow \infty$$

where $$HM(x_1, x_2, \ldots\ldots\ldots, x_N) = \left(\frac{1}{N}\sum_{i=1}^{N} x_i^{-1}\right)^{-1}$$

Accordingly, $HM(x_1, x_2, \ldots\ldots\ldots, x_N)$ can also be regarded as a measure of μ and consequently a measure of central tendency of $x_1, x_2, \ldots\ldots\ldots, x_N$.

AGM as a Measure of Central Tendency

Since each of a_n & g_n is approximate value of μ,

$$a_0 = \mu + \delta_0 \quad \& \quad g_0 = \mu + \xi_0 \quad , \text{ for real numbers } \delta_0 \ \& \ \xi_0 .$$

This implies, $\delta_0 > \xi_0$ since $a_0 > g_0$

By the same logic, $\mathbf{a}_{n+1} = \mu + \delta_{n+1}$ & $\mathbf{g}_{n+1} = \mu + \xi_{n+1}$, for real numbers δ_{n+1} & ξ_{n+1}

Since $\mathbf{a}_{n+1}$ is the AM of a_n & g_n

therefore, $a_n > \mathbf{a}_{n+1} > g_n$

which implies, $\delta_n > \delta_{n+1}$ i.e. the sequence $\{\delta_n\}$ is decreasing.

This implies, $\delta_n < \delta_0$ i.e. $\delta_n < a_0 - \mu$

Also, $\delta_0 > \xi_0 \Rightarrow \delta_0 > g_0 - \mu$

Hence, $g_0 - \mu < \delta_n < a_0 - \mu$ i.e. the sequence $\{\delta_n\}$ is bounded.

Hence, the sequence $\{\delta_n\}$ is convergent and converges to a point δ_{AG} in $(g_0 - \mu \ , \ a_0 - \mu)$.

Accordingly, $\{a_n\}$ & $\{g_n\}$ and hence $AGM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ converge to the point $\mu + \delta_{AG}$.

Therefore, $AGM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be regarded as a measure of μ and consequently a measure of central tendency of $x_1, x_2, \ldots\ldots\ldots\ldots, x_N, x_N$ with deviation δ_{AG} in $(g_0 - \mu \ , \ a_0 - \mu)$.

AHM as a Measure of Central Tendency

Since each of a'_n & h'_n is approximate value of μ ,

$$a'_0 = \mu + \delta'_0 \ \ \& \ \ h'_0 = \mu + e'_0 , \text{ for real numbers } \delta'_0 \ \& \ e'_0 .$$

This implies, $\delta'_0 > e'_0$ since $a'_0 > h'_0$

By the same logic,

$a'_{n+1} = \mu + \delta'_{n+1}$ & $h'_{n+1} = \mu + e'_{n+1}$, for real numbers δ'_{n+1} & e'_{n+1} .

Since a'_{n+1} is the AM of a'_n & h'_n ,

therefore, $a'_n > a'_{n+1} > h'_n$

which implies, $\delta'_n > \delta'_{n+1}$ i.e. the sequence $\{\delta'_n\}$ is decreasing.

Moreover, $h_0 - \mu < \delta'_n < a_0 - \mu$ i.e. the sequence $\{\delta'_n\}$ is bounded.

Hence, the sequence $\{\delta'_n\}$ is convergent and converges to a point δ_{AH} in $(h_0 - \mu \ , \ a_0 - \mu)$.

Accordingly, $\{a'_n\}$ & $\{h'_n\}$ and hence $AHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ converge to the point $\mu + \delta_{AH}$.

Therefore, $AHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be regarded as a measure of μ and consequently a measure of central tendency of $x_1, x_2, \ldots\ldots\ldots\ldots, x_N$ with deviation δ_{AH} in $(h_0 - \mu \ , \ a_0 - \mu)$.

GHM as a Measure of Central Tendency

Since each of $g''_0 = g_0$ & $h''_0 = h_0$ is approximate value of μ ,

$g''_0 = \mu + \xi_0$ & $h''_0 = \mu + e_0$, for real numbers ξ_0 & e_0 .

This implies, $\xi_0 > e_0$ since $g''_0 > h''_0$

By the same logic,

$g''_{n+1} = \mu + \xi''_{n+1}$ & $h''_{n+1} = \mu + e''_{n+1}$, for real numbers ξ''_{n+1} & e''_{n+1} .

Since g''_{n+1} is the GM of g''_n & h''_n ,

therefore, $g''_n > g''_{n+1} > h''_n$

which implies, $\xi''_n > \xi''_{n+1}$ i.e. the sequence $\{\xi''_n\}$ is decreasing .

Moreover, $h_0 - \mu < \xi''_n < g_0 - \mu$ i.e. the sequence $\{\xi''_n\}$ is bounded.

Hence, the sequence $\{\xi''_n\}$ is convergent and it converges to a point ξ_{GH} in $(h_0 - \mu \ , \ g_0 - \mu)$.

Accordingly, $\{g''_n\}$ & $\{h''_n\}$ and hence $GHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ converge to the point $\mu + \xi_{GH}$.

Therefore, $GHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be regarded as a measure of μ and consequently a measure of central tendency of $x_1, x_2, \ldots\ldots\ldots\ldots, x_N$ with deviation ξ_{GH} in $(h_0 - \mu \ , \ g_0 - \mu)$.

AGHM as a Measure of Central Tendency

In this case as earlier,

$a'''_0 = \mu + \delta_0$, $g'''_0 = \mu + \xi_0$ & $h'''_0 = \mu + e_0$, for real numbers δ_0, ξ_0 & e_0 .

This implies, $\delta_0 > \xi_0 > e_0$ since $a'''_0 > g'''_0 > h'''_0$

By the same logic, $a'''_{n+1} = \mu + \delta'''_{n+1}$, $g'''_{n+1} = \mu + \xi'''_{n+1}$ & $h'''_{n+1} = \mu + e'''_{n+1}$,

for real numbers δ'''_{n+1} , ξ''_{n+1} & e''_{n+1} .

Since a'''_{n+1} is the AM of a'''_n , g''_n & h''_n ,

therefore, $a'''_n > a'''_{n+1} > h'''_n$

which implies, $\delta'''_n > \delta'''_{n+1}$ i.e. the sequence $\{\delta'''_n\}$ is decreasing .

Moreover, $h_0 - \mu \ < \ \delta'''_n < \ a_0 - \mu$ i.e. the sequence $\{\delta'''_n\}$ is bounded.

Hence, the sequence $\{\delta'''_n\}$ is convergent and it converges to a point δ_{AGH} in $(h_0 - \mu \ \ , \ a_0 - \mu)$.

Accordingly, the three sequences $\{a'''_n\}$, $\{g'''_n\}$ & $\{h'''_n\}$ and hence $AGHM\ (x_1 , x_2 , \ \ldots\ldots\ldots , x_N)$ converge to the point $\mu + \delta_{AGH}$.

Therefore, $AGHM\ (x_1 , x_2 , \ \ldots\ldots\ldots , x_N)$ can be regarded as a measure of μ and consequently a measure of central tendency of x_1 , x_2 , ... , x_N with deviation δ_{AGH} in $(h_0 - \mu \ \ , \ a_0 - \mu)$.

4. Numerical Example – Application to Numerical data:

The following table (**Table – 1)** shows the observed data on the sex ratio population (state-wise) in India in 2011, as published in "Census Report" by Register General of India:

Table – 1

State	**Value of the Ratio** **Male / Female (M / F)**	**Value of the Ratio** **Female / Male (F / M)**
Jammu & Kashmir	1.12541385341251118522736516716 83	0.888562013847414610169889688708 75
Himachal Pradesh	1.029308880492643661375179625680 9	0.971525670235531278711199403309 66
Punjab	1.11718611741734676457868601138	0.895106002849147834295857123194 05
Chandigarh	1.222996838582353771270064260394 7	0.817663601779345334557221658690 15
Uttarakhand	1.038244573780559395649486240226 6	0.963164195849057558595913054157 92
Haryana	1.138149917920019755871940386926 3	0.878618874592118673847146598073
Delhi	1.152130440997280342639650848042 1	0.867957276725023661097861588641 61
Rajasthan	1.07738651846931155871485787988 4	0.928172000352057637089615236388 45
Uttar Pradesh	1.095966676649691119433130367547 4	0.912436501314934239888377267683 73
Bihar	1.089456968149864430460910339644 9	0.917888479522250543621073943243 87
Sikkim	1.123694379615105023529861881623 8	0.889921688798093292475314947225 06
Arunachal Pradesh	1.065834596119824130543508282137 6	0.938231882921144342720111162160 04
Nagaland	1.074221080187408332311163250521 8	0.930907071592320882565639550714 44
Manipur	1.015084588853576892029963138791 2	0.985139574554458336171768667288 57
Mizoram	1.024862189430247643794510461054 1	0.975740943819900997408789946321 08
Tripura	1.041585604329103921499982495536 4	0.960074712864441286060000768255 68

Meghalaya	1.0113724418785172369610123540989	0.98875543626896326127874988604615
Assam	1.0441048168517855831597956077024	0.95775824788858682201128358123932
West Bengal	1.0526667948213744061675457587868	0.94996821873695430584361430969287
Jharkhand	1.0543346515488809532602154750904	0.9484654597389357492757813425208
Odisha	1.0216767277963741786589610810708	0.97878318336258074151514020087369
Chhattisgarh	1.0094862433659738915914763831542	0.99060289981333128651017560729672
Madhya Pradesh	1.0741921997293521487367677330733	0.93093209972289388478334723747063
Gujarat	1.0878399216924771607664276945985	0.9192528974705997791133158851059
Daman & Diu	1.6170787338884943945947109074086	0.61839907918110990612171575704752
Dadra & Nagar Haveli	1.29217267204182755470193199021	0.77389037985136251032204789430223
Maharashtra	1.0759593940486569345112623151307	0.92940310343605596519523288750508
Andhra Pradesh	1.0072027731513157131279371653056	0.99284873578258743089946488568226
Karnataka	1.0278146308628600560795309711955	0.97293808627776643762353811714322
Goa	1.0274323920462048498411882041409	0.97330005141109938577265470682144
Lakshadweep	1.0565550239234449760765550239234	0.94647223983334842858436735802916
Kerala	0.92224729321594561234305382426448	1.0843078720382305015931455433978
Tamil Nadu	1.0035802105886977594941050244168	0.99643256159206485698216349975338
Pondicherry	0.96391330758747454527714567183158	1.0374376949964980220763382208646
Andaman & Nicobar	1.1415846041303246862866467840864	0.87597537351321775906857066806002
India	1.0607325851848778252519531570732	0.94274467850509882664736426425148

Central Tendency of the Ratio M / F:

From the observed values on the ratio **M / F** in **Table – 1** it has been obtained that

AM of the ratio **M / F** = 1.0835068016450523020161865887443 ,

GM of the ratio **M / F** = 1.0784172361960199316030087704149

& *HM* of the ratio **M / F** = 1.0740468088974845410059550737324

Computation central tendency of the Ratio **M / F** ***by AGM***

Table – 2

N	Value of a_n	Value of g_n
0	1.0835068016450523020161865887443	1.0784172361960199316030087704149
1	<u>1.0809</u>620189205361168095976795796	<u>1.0809</u>590234738995349229858558766

2	1.0809605211972178258662917677281	1.0809605211961802416556912352376
3	1.0809605211966990337609915014829	1.0809605211966990337609913769893
4	1.0809605211966990337609914392361	1.0809605211966990337609914392361

Central tendency of the Ratio **M / F** by *AGM* = 1.0809605211966990337609914392361

Computation central tendency of the Ratio* M / F *by AHM

Table – 3

n	Value of a'_n	Value of h'_n
0	1.0835068016450523020161865887443	1.0740468088974845410059550737324
1	1.0787768052712684215110708312384	1.0787560661660274789282541031017
2	1.0787664357186479502196624671701	1.0787664356189714883012948072843
3	1.0787664356688097192604786372272	1.078766435668809719258176146917
4	1.0787664356688097192593273920721	1.0787664356688097192593273920721

Central tendency of the Ratio **M / F** by *AHM* = 1.0787664356688097192593273920721

Computation central tendency of the Ratio* M / F *by GHM

Table – 4

N	Value of g''_n	Value of h''_n
0	1.0784172361960199316030087704149	1.0740468088974845410059550737324
1	1.0762298040829291529618372458428	1.0762275856236790442484768081998
2	1.076228694852732477365646095545	1.076228694852160856126135467676
3	1.0762286948524466667458907436596	1.0762286948524466667458907057087
4	1.0762286948524466667458907246841	1.0762286948524466667458907246841

Central tendency of the Ratio **M / F** by *GHM* = 1.0762286948524466667458907246841

Computation central tendency of the Ratio* M / F *by AGHM

Table – 5

N	Term of sequence	Values of a'''_n , g'''_n & h'''_n
1	a'''_1	1.07865694891285225820838347763 05
	g'''_1	1.0786500232826463705959730067094
	h'''_1	1.0786430991879167734854095348694
2	a'''_2	1.0786500237944718007632553397364
	g'''_2	1.0786500237796527462927936308012
	h'''_2	1.0786500237648336918293636455239
3	a'''_3	1.0786500237796527462951375386872
	g'''_3	1.0786500237796527462950696747305
	h'''_3	1.0786500237796527462950018107739
4	a'''_4	1.0786500237796527462950696747305
	g''_4	1.0786500237796527462950696747305
	h'''_4	1.0786500237796527462950696747305

Central tendency of the Ratio **M / F** by *AGHM* = 1.0786500237796527462950696747305

Central Tendency of the Ratio F / M:

From the observed values on **F / M** in **Table – 3** it has been obtained that

AM of the ratio **F / M** = 0.9310581175009550726813265197974 ,

GM of the ratio **F / M** = 0.9272848823590516878417869187 2109

& *HM* of the ratio **F / M** = 0.9229291394218599224261917978 4686

Computation central tendency of the Ratio the ratio **F / M *by AGM***

Table – 6

n	Value of a_n	Value of g_n
0	0.9310581175009550726813265197974	0.9272848823590516878417869187 2109
1	0.9291714999300033802615567192 5924	0.9291695846056914749693624331455
2	0.9291705422678474276154595762 0237	0.9291705422673539139044535825 0697

3	0.92917054226760067075995657935467	0.92917054226760067075995654658945
4	0.92917054226760067075995656297206	0.92917054226760067075995656297206

Central tendency of the Ratio **M / F** by ***AGM*** = 0.92917054226760067075995656297206

Computation central tendency of the Ratio the ratio **F / M** ***by AHM***

Table – 7

n	Value of a'_n	Value of h'_n
0	0.9310581175009550726813265197974	0.92292913942185992242619179784686
1	0.92699362846140749755375915882213	0.92697580733443813334996246257971
2	0.92698471789792281545186081070092	0.92698471781227076522756233102558
3	0.92698471785509679033971157086325	0.92698471785509679033773303940364
4	0.92698471785509679033872230513345	0.92698471785509679033872230513345

Central tendency of the Ratio **M / F** by ***AHM*** = 0.92698471785509679033872230513345

Computation central tendency of the Ratio the ratio **F / M** ***by GHM***

Table – 8

n	Value of g''_n	Value of h''_n
0	0.92728488235905168784178691872109	0.92292913942185992242619179784686
1	0.92510444733259184113762549477893	0.92510188378183173522959886699897
2	0.92510316555632380715291216365606	0.92510316555543582612221299877169
3	0.92510316555587981663756247467031	0.92510316555587981663756236812669
4	0.9251031655558798166375624213985	0.9251031655558798166375624213985

Central tendency of the Ratio **M / F** by *GHM* = 0.9251031655558798166375624213985

Computation central tendency of the Ratio the ratio **F / M** ***by AGHM***

Table – 9

N	Term of sequence	Values of a'''_n , g'''_n & h'''_n
1	a'''_1	0.927090713093955560983101745455 12
	g'''_1	0.92708476189219240096038081993126
	h'''_1	0.92707880944712926028363970338239
2	a'''_2	0.92708476147775907407570742292292
	g'''_2	0.92708476146502230193490281506759
	h'''_2	0.92708476145228552978840450686974
3	a'''_3	0.92708476146502230193300491495342
	g'''_3	0.92708476146502230193294658682228
	h'''_3	0.927084761465022301932888825869 11
4	a'''_4	0.92708476146502230193294658682227
	g''_4	0.92708476146502230193294658682227
	h'''_4	0.92708476146502230193294658682227

Central tendency of the Ratio **M / F** by ***AGHM*** = 0.92708476146502230193294658682227

5. Discussions and Conclusion:

The numerical findings can be summarized as follows (**Table – 10**):

Table – 10

Measure	Central tendency of sex ratio	
	M / F	**F / M**
AM	1.08350680164505230201618658874 43	0.93105811750095507268132651979 74
GM	1.07841723619601993160300877041 49	0.92728488235905168784178691872109
HM	1.07404680889748454100595507373 24	0.92292913942185992242619179784686
AGM	1.08096052119669903376099143923 61	0.92917054226760067075995656297206
AHM	1.07876643566880971925932739207 2	0.92698471785509679033872230513345
GHM	1.07622869485244666674589072468 41	0.92510316555587981663756242139 85
AGHM	1.07865002377965274629506967473 05	0.92708476146502230193294658682227

If μ is the central tendency of

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$$x_1\ , x_2\ ,\ \ldots\ldots\ldots\ldots\ , x_N$$

then the central tendency of

$$x_1^{\,-1},\ x_2^{\,-1}\ ,\ \ldots\ldots\ldots\ldots\ ,\ x_N^{\,-1}$$

should logically be $\mu^{\,-1}$.

In the example, it has been found that the value of central tendency of the ratio **M / F**, obtained by *AHM*, is

1.0787664356688097192593273920721

and the value of central tendency of the ratio **F / M**, obtained by *AHM*, is

These two values are reciprocals each other i.e.

(1.0787664356688097192593273920721)$^{-1}$ = 0.92698471785509679033872230513345

& (0.92698471785509679033872230513345)$^{-1}$ = 1.0787664356688097192593273920721

Thus, *AHM* can be regarded as a valid measure of central tendency of data of ratio type.

Similarly, it is also observed in that the *AGHM* of the ratio **M / F** is

1.0786500237796527462950696747305

and of the ratio **F / M** is

0.92708476146502230193294658682227

which are also reciprocals each other i.e.

(1.0786500237796527462950696747305)$^{-1}$ = 0.92708476146502230193294658682227

& (0.92708476146502230193294658682227)$^{-1}$ = 1.0786500237796527462950696747305

Accordingly, *AGHM* can also be regarded as a valid measure of central tendency of data of ratio type. Moreover, *GM* has also been found to satisfy this property which implies that *GM* can also be regarded as a valid measure of central tendency of data of ratio type.

However, it is found that this property is not satisfied by the other four measures of central tendency of data of ratio type which implies that these four measures cannot be logically accepted as valid measures of central tendency of data of ratio type.

It is to be noted that the GM of

AM of the Ratio **M / F** & *HM* of the Ratio **M / F** is found to be 1.0787664356688097192593273920721 which is nothing but the AHM of the observed values of the Ratio **M / F**.

Similarly, the GM of

AM of the Ratio **F / M** & *HM* of the Ratio **F / M**

is found to be 0.92698471785509679033872230513345 which is nothing but the *AHM* of the observed values of the Ratio **F / M**.

Thus, *AHM* of the observed values can be regarded as the *GM* of *AM* of the observed values and *HM* of observed values. In general, $AHM(x_1, x_2, \ldots, x_N)$ can be defined as the *GM* of $AM(x_1, x_2, \ldots, x_N)$ and $HM(x_1, x_2, \ldots, x_N)$ in the instant case. However, it is to be established for general case.

Finally, from the meaning of research [Chakrabarty (2018g , 2018h , 2019a)], it can be concluded that the extraction of information of *GM*, *AHM* & *AGHM* as valid measures of central tendency of data of ratio type can be regarded as research findings carrying fundamental importance and high significance in the theory of analysis of data specially of measure of central tendency of data.

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