Recent Developments on Representation of Experimental Data by Non-polynomial Curve

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Abstract

Recently some studies have been made on representing numerical data on a pair of variables by some standard non-polynomial curves namely exponential curve, modified exponential curve, logistic curve, Makeham's curve etc. in connection with the development of some formula/method, more convenient than the existing ones, of interpolation. This paper is based on a brief review on the recent developments of the methods of representing numerical data on a pair of variables by these non-polynomial curves along with their application in real data.

Keywords : Pair of variables , numerical data, mathematical representation, non-polynomial curve

1 Introduction

Observations or data, collected from experiment or survey, normally suffer from various types of errors/causes which can be broadly divided into two types namely (1) Assignable error/cause that is avoidable/controllable & (2) Chance error/cause that is unavoidable / uncontrollable [Chakrabarty, 2014a,b,c,d,e,f,g,h, 2015a,b,c,d,e,f]. Even if all the assignable causes of error are controlled or eliminated, observations still do not become free from error. Each of them still suffers from some error which occurs due to some unknown and unintentional cause that is nothing but the chance cause. Consequently the findings obtained by analyzing the observations which are free from the assignable errors are also subject to errors due to the presence of chance error in the observations. Determination of constant(s) associated to mathematical model(s), in different situations, based on the observations is also subject to error due to the same reason.

A number of mathematical models have been identified for describing the association of chance error(s) in determining constant(s) in some distinct situations where observations/data are of measurement type [Chakrabarty, 2014g, 2016e,j, 2017c].

There are two broad aspects of statistical determination of parameters involved in the respective models describing the dependence of the dependent variable on the independent variable(s). One of them is based on the basic philosophy behind statistics [Chakrabarty, 2018g,h, 2019b], which consists of determining the parameter(s) from numerical data compromising with some degree of error in findings. Several

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statistical methods have already been developed for determination (estimation in statistical literature) of such parameter(s) which are available in the standard literatures in statistics. However, existing statistical methods of estimation cannot normally yield error free estimate(s) of parameters [Chakrabarty, 2014g, 2016e,j]. The same fact happens in the case of some recently developed methods of estimation [Chakrabarty, 2011, 2014a, 2016c,d,i; Chakrabarty and Dutta, 2007; Chakrabarty and Rahman, 2007, 2008]. Recently, some studies have been made on attempting of determining error free estimates of such parameter(s) [Chakrabarty, 2014b,f,g, 2015c,d,f, 2017b,c, 2018e, 2019a,d, 2020d] along with attempting of application in the case of real data [Chakrabarty, 2015b, 2016a, 2019h,i] . These studies have been done on the basis of various measures of average namely the three Pythagorean means (namely arithmetic mean, geometric mean & harmonic mean), median, generalized mean and others [Chakrabarty, 2016g, 2017c, 2018a,c,d,e,f, 2019c,e,f,g, 2020a,b,c,d,e, 2021a,b,c,d,e,f,g,h].

Standard methods of statistical representation of numerical data like least squares method, orthogonal polynomial method etc. are available in the standard literature of statistics. Some studies, extension in nature, have recently been done on statistical representation of numerical data by some special mathematical curves namely linear curve, quadratic curve, exponential curve etc. [Chakrabarty, 2011, 2014a, 2016c,d, 2017a; Chakrabarty and Dutta, 2007; Chakrabarty and Rahman, 2007, 2008; Rahman and Chakrabarty, 2009, 2011, 2015a,b,c,d,e,f].

Later on, some studies have been made on the representation of numerical data on a pair of variables by suitable mathematical equation/mathematical curve. In those studies [Chakrabarty, 2016a,b,f,g,h,i, 2017d,e, 2018b; Das and Chakrabarty, 2016a,b,c,d,e,f, 2017a,b,c,d, 2020] , some formulas/methods have been developed for representing numerical data on a pair of variables by polynomial curve in connection with the development of some more convenient formula/method of interpolation which is a technique of estimating approximately the value of the dependent variable corresponding to a value of the independent variable lying between its two extreme values on the basis of the given values of the independent and the dependent variables [Bathe and Wilson, 1976; Hummel, 1947; Wisniewski, 1930]. The formulas developed are based on usual algebraic operation, forward difference operation, backward difference operation, divided difference operation, backward divided difference operation, difference and ratio operation and backward difference & ratio operation [Bathe and Wilson, 1976; Conte and de Boor, 1980; Dokken and Lyche, 1979; Gerald and Patrick, 1994; Jordan and Jordán, 1965; Lee, 1989; Vertesi, 1990; Whittaker and Robinson, 1967], while the methods developed are based on matrix inversion by Cayley-Hamilton Theorem, matrix inversion by Gauss Jordan method and matrix inversion by elementary column transformation. Recently some studies have been made on representing numerical data on a pair of variables by some standard non-polynomial curves namely exponential curve, modified exponential curve, logistic curve, Makeham's curve etc. in connection with the development of some formula/method, more convenient than the existing ones, of interpolation. This paper is based on a brief review on the recent developments of the methods of representing numerical data on a pair of variables by these non-polynomial curves along with their application in real data.

2 Method of Representation of Numerical Data

Methods of representation of numerical data on a pair of variables by some standard non-polynomial curves, as mentioned above, have been discussed below:

2.1 Exponential Curve

The Exponential curve is of the form

$$y = ab^x \tag{1}$$

Eq. 1 implies,

$$\log y = \log a + x \log b \tag{2}$$

Since there are two parameters in the exponential curve, two equations are necessary for determining the values of the parameters and accordingly two sets of values the pair of variables are necessary.
Let y_0, y_1 be the values of y corresponding to the values x_0, x_1, of x respectively.Then the points (x_0, y_0) and (x_1, y_1) lie on the Eq. 1 and hence on Eq. 2.
Accordingly,

$$\log y_0 = \log a + x_0 \log b$$

$$\log y_1 = \log a + x_1 \log b$$

From which one can obtain that

$$b = \frac{\log y_1 - \log y_0}{x_1 - x_0} = \text{antilog}\left(\frac{\Delta \log y_0}{\Delta x_0}\right) \tag{3}$$

$$a = \text{antilog}\left[\log y_0 - x_0\left(\frac{\Delta y_0}{\Delta x_0}\right)\right] = \text{antilog}\left[\log y_1 - x_1\left(\frac{\Delta y_0}{\Delta x_0}\right)\right] \tag{4}$$

2.2 Modified Exponential Curve

The modified exponential curve is of the form

$$y = a + bc^x \tag{5}$$

where, a,b and c are parameters.
Since there are three parameters in the modified exponential curve, three equations are necessary for determining the values of the parameters and accordingly three sets of values the pair of variables are necessary.
Let y_0, y_1,y_2 be the values of y corresponding to the values x_0, x_1,x_2 of x respectively so that three points (x_0, y_0), $(x_1, y_1)$and (x_2, y_2) lie on the Eq. 1. Then,
$y_0 = a + bc^{x_0}$, $y_1 = a + bc^{x_1}$, $y_2 = a + bc^{x_2}$
i.e., $\Delta y_0 = b(c^{x_1} - c^{x_0})$ & $\Delta y_1 = b(c^{x_2} - c^{x_1})$
If x_0,x_1,x_2 are equally spaced then,
$x_1 - x_0 = x_2 - x_1 = h$,
i.e., $x_1 = x_0 + h$, & $x_2 = x_0 + 2h$
This means,
$y_0 = a + bc^{x_0}$, $y_1 = a + bc^{x_0+h}$, & $y_2 = a + bc^{x_0+2h}$
Accordingly,
$\Delta y_0 = bc^{x_0}(c^h - 1)$ & $\Delta y_1 = bc^{x_1}(c^h - 1)$
which implies,

$$\frac{\Delta y_1}{\Delta y_0} = c^h$$

, i.e.,

$$c = \text{antilog}\left[\frac{1}{h}\log\left(\frac{\Delta y_1}{\Delta y_0}\right)\right] \tag{6}$$

and consequently,

$$b = \frac{\Delta y_0}{c^{x_0}(c^h - 1)} = \frac{\Delta y_1}{c^{x_1}(c^h - 1)} \tag{7}$$

where c is given by the Eq. 6. &

$$a = y_0 - bc^{x_0} = y_1 - bc^{x_1} = y_2 - bc^{x_2} \tag{8}$$

where b and c are given by the Eq. 6 and Eq. 7 respectively.

2.3 Logistic Curve

The logistic curve is the form

$$y = \frac{A}{B + C^x} \tag{9}$$

where, A,B and C are parameters.
In this case also there are three parameters so that three equations are necessary for determining the values of the parameters and accordingly three sets of values the pair of variables are necessary.
As earlier, let y_0, y_1,y_2 be the values of y corresponding to the values x_0, x_1,x_2, of x respectively so that three points (x_0, y_0),(x_1, y_1) and (x_2, y_2) lie on the curve described by Eq. 9. Then

$$y_0 = \frac{A}{B + C^{x_0}}\ , y_1 = \frac{A}{B + C^{x_1}}\ \&\ y_2 = \frac{A}{B + C^{x_2}},$$

so that

$$\frac{1}{y_0} = \frac{B}{A} + \frac{1}{A}c^{x_0}\ , \frac{1}{y_1} = \frac{B}{A} + \frac{1}{A}c^{x_1}\ \&\frac{1}{y_2} = \frac{B}{A} + \frac{1}{A}c^{x_2}$$

i.e.,

$$\Delta\left(\frac{1}{y_0}\right) = \frac{1}{A}\left(c^{x_1} - c^{x_0}\right)\ \&\ \Delta\left(\frac{1}{y_1}\right) = \frac{1}{A}\left(c^{x_2} - c^{x_1}\right)$$

If x_0, x_1, x_2 are equally spaced then

$$x_1 - x_0 = x_2 - x_1 = h$$

$$\text{i,e,}\ \ x_1 = x_0 + h\ \&\ x_2 = x_0 + 2h$$

this means,

$$y_0 = \frac{A}{B + C^{x_0}},\ \ y_1 = \frac{A}{B + C^{x_0+h}}\ \ \&\ \ y_2 = \frac{A}{B + C^{x_0+2h}}$$

$$\text{i.e.,}\ \frac{1}{y_0} = \frac{B + C^{x_0}}{A},\ \ \frac{1}{y_1} = \frac{B + C^{x_0+h}}{A}\ \ \&\ \ \frac{1}{y_2} = \frac{B + C^{x_0+2h}}{A}$$

Accordingly,

$$\Delta\left(\frac{1}{y_0}\right) = \frac{1}{A}C^{x_0}(c^h - 1)$$

$$\&\ \ \Delta\left(\frac{1}{y_0}\right) = \frac{1}{A}C^{x_1}(c^h - 1) = \frac{1}{A}C^{x_0+h}(c^h - 1) = \frac{c^h}{A}C^{x_0}(c^h - 1)$$

which implies,

$$\frac{\Delta\frac{1}{y_1}}{\Delta\frac{1}{y_0}} = c^h$$

$$C = \text{antilog}\left[\frac{1}{h}\log\left\{\frac{\Delta\left(\frac{1}{y_1}\right)}{\Delta\left(\frac{1}{y_0}\right)}\right\}\right] \tag{10}$$

and consequently,

$$A = \frac{c_0^x(c^h-1)}{\Delta\left(\frac{1}{y_0}\right)} = \frac{c_1^x(c^h-1)}{\Delta\left(\frac{1}{y_1}\right)} \tag{11}$$

where C is given by Eq. 10.

$$B = \frac{A}{y_0} - c^{x_0} = \frac{A}{y_1} - c^{x_1} = \frac{A}{y_2} - c^{x_2} \tag{12}$$

where A and C are given by Eq. (11) and Eq. 10.

2.4 Makeham's Curve

The Makeham's curve is of the form

$$y = ab^x c^{d^x} \tag{13}$$

where a, b, c and d are parameters.
In this case there are four parameters so that four equations are necessary for determining the values of the parameters and accordingly four sets of values the pair of variables are necessary.
As earlier, let y_0, y_1,y_2,y_3 be the values of y corresponding to the values x_0, x_1,x_2,x_2 of x so that the four points $(x_0, y_0)(x_1, y_1), (x_2, y_2), (x_3, y_3)$ lie on the curve describe by Eq. 13.
Then,

$$y_0 = ab^{x_0}c^{d^{x_0}}, y_1 = ab^{x_1}c^{d^{x_1}}, y_2 = ab^{x_2}c^{d^{x_2}}, y_3 = ab^{x_3}c^{d^{x_3}}$$

i.e.,

$$\log y_0 = \log a + x_0 \log b + d^{x_0}\log c$$
$$\log y_1 = \log a + x_1 \log b + d^{x_1}\log c$$
$$\log y_2 = \log a + x_2 \log b + d^{x_2}\log c$$
$$\log y_3 = \log a + x_3 \log b + d^{x_3}\log c$$
$$\log y_4 = \log a + x_4 \log b + d^{x_4}\log c$$

If $x_0, x_1, x_2, x_3$are equally spaced then,

$$x_1 - x_0 = x_2 - x_1 = x_3 - x_2 = h$$

$$i.e., x_1 = x_0 + h, x_2 = x_0 + 2h, x_3 = x_0 + 3h,$$

so that

$$\Delta\log y_0 = h\log b + (d^h - 1)\log cd^{x_0}$$
$$\Delta\log y_1 = h\log b + (d^h - 1)\log cd^{x_1}$$
$$\Delta\log y_2 = h\log b + (d^h - 1)\log cd^{x_2}$$

This implies,

$$\frac{\Delta^2\log y_1}{\Delta^2\log y_0} = \frac{d^{x_1}}{d^{x_0}} = \frac{d^{x_0+h}}{d^{x_0}} = \frac{d^{x_0}d^h}{d^{x_0}}$$

If $h = 1$,

$$d = \frac{\Delta^2\log y_1}{\Delta^2\log y_0} \tag{14}$$

Table 1: Total Population of India

Year	1951	1961	1971	1981	1991	2011
x	0	1	2	3	4	5
Number of Person	361088090	439234771	548159652	683329097	846302688	1210193422

Also,

$$\Delta^2 \log y_0 = (d-1)^2 \log c$$

Which implies,

$$c = \text{antilog}\left\{\frac{\Delta^2 \log y_0}{(d-1)^2}\right\} (\text{If } \ x_0 = 0) \tag{15}$$

and consequently,

$$b = \text{antilog}\left[\Delta \log y_0 - \frac{\Delta^2 \log y_0}{(d-1)}\right] \tag{16}$$

$$a = \text{antilog}\left[\Delta \log y_0 - \frac{\Delta^2 \log y_0}{(d-1)^2}\right] \tag{17}$$

3 Application to Numerical Data

The Table 1 shows the data on total population of India corresponding to the years from 1951 onwards:

3.1 Representation by Exponential Curve

Let us first represent the total populations corresponding to the years 1951 and 1961 by the exponential curve described by Eq. 1

After computations of the values of the parameters a and b (which have been found to be 1.21641999047 and 361088090 respectively), the form of the exponential curve that can represent the total populations corresponding to the years 1951 and 1961 has been found as

$$y = 361088090 \times 1.21641999047^x$$

The forms of exponential curves, obtained, for representing the total population of India corresponding to the other pairs of years have been obtained have been shown in the Table 2

3.2 Representation by Modified Exponential Curve

In order to represent the total populations corresponding to the years 1951, 1961 and 1971 by modified exponential curve let us construct the Table 3 After computations it is obtained that

$$c = 1.3938516595, \ b = 198416533.52 \ \& \ a = 162671556.48$$

Therefore, the modified exponential curve that can represent the given data is

$$y = 162671556.48 + 198416533.52 \times 1.3938516595^x$$

Similarly, the modified exponential curves representing the total populations corresponding to the other years obtained have been shown in Table 4

Table 2: Exponential curve representing total population of India

Years (x=0,1)	Curve representing total population
1951 , 1961	$y = 361088090 \times 1.21641999047^x$
1961 , 1971	$y = 439234771 \times 1.247987837465^x$
1971 , 1981	$y = 548159652 \times 1.246587731342^x$
1981 , 1991	$y = 683329097 \times 1.238499416627^x$
1991 , 2001	$y = 846302688 \times 1.213531826807^x$
2001 , 2011	$y = 1027015247 \times 1.178359742501^x$
1951 , 1971	$y = 361088090 \times 1.518077353367^x$
1961 , 1981	$y = 439234771 \times 1.555726327049^x$
1971 , 1991	$y = 548159652 \times 1.543898178043^x$
1981 , 2001	$y = 683329097 \times 1.502958459560^x$
1991 , 2011	$y = 846302688 \times 1.429977050953^x$
1951 , 1981	$y = 361088090 \times 1.892416603937^x$
1961 , 1991	$y = 439234771 \times 1.926766148483^x$
1971 , 2001	$y = 548159652 \times 1.873569576405^x$
1981 , 2011	$y = 683329097 \times 1.771025743397^x$
1951 , 1991	$y = 361088090 \times 2.343756859993^x$
1961 , 2001	$y = 439234771 \times 2.338192043999^x$
1971 , 2011	$y = 548159652 \times 2.207738963611^x$
1951 , 2001	$y = 361088090 \times 2.844223543900^x$
1961 , 2011	$y = 439234771 \times 2.755231374885^x$
1951 , 2011	$y = 361088090 \times 3.351518522806^x$

3.3 Representation by Logistic Curve

In order to represent the total populations corresponding to the years 1951, 1961 1971 by logistic curve let us construct the Table 5

After computations it is obtained that

$$C = 0.9181690619,\ A = 166079821.3578811825 \text{ and } B = -0.5400573269$$

Therefore, the logistic curve that can represent the given data is

$$y = 166079821.3578811825/(-0.5400573269 + 0.9181690619^x$$

Similarly, the modified exponential curves representing the total populations corresponding to the other years obtained have been shown in the Table 6

3.4 Representation by Makeham's Curve

In order to represent the total populations corresponding to the years 1951, 1961, 1971 and 1981 by Makeham's curve let us construct the Table 7 After computations it is obtained that

$$d = -0.043813525170,\ \ c = 1.023793397619 \ \& \ a = 352696247.934125408029$$

Table 3:

Year	x	Number population $f(x) = y_i$	Δy_i	$\frac{\Delta y_1}{\Delta y_0}$	$\log\left(\frac{\Delta y_i}{\Delta y_0}\right)$
1951	0	361088090	78146681	1.393851659547	0.332070893151
1961	1	439234771	108924881		
1971	2	548159652			

Table 4: Modified exponential curve representing total population of India

Years $x = (0, 1, 2)$	Curve Representing total population
1951 , 1961 , 1971	$y = 162671556.48 + 198416533.52 \times 1.3938516595^x$
1961 , 1971 , 1981	$y = -12844741.85 + 452079512.85 \times 1.2409418651^x$
1971 , 1981 , 1991	$y = -108964607.87 + 657124259.87 \times 1.2056984549^x$
1981 , 1991 , 2001	$y = -813961579.52 + 1497290676.52 \times 1.1088456594^x$
1991 , 2001 , 2011	$y = -12398675095.11 + 13244977783.11 \times 1.0136438552^x$
1951 , 1981 , 2011	$y = -146377372.04 + 507465462.04 \times 1.6350008643^x$

Table 5:

Year	X	$\frac{1}{y_i}$	$\Delta\frac{1}{y_i}$	$\frac{\Delta\frac{1}{y_i}}{\Delta\frac{1}{y_0}}$
1951	0	0.000000002769407321	−0.000000000492720533	0.9181690619
1961	1	0.000000002276686788	−0.0000000004524007496	
1971	2	0.0000000018242860384		

Table 6: Logistic curve representing total population of India

Years $x = (0, 1, 2)$	Curve Representing total population
1951 , 1961 , 1971	$y = \frac{166079821.3578811825}{-0.5400573269+(0.9181690619)^x}$
1961 , 1971 , 1981	$y = \frac{447256612.3705644717}{0.0182632202+(0.7976607733)^x}$
1971 , 1981 , 1991	$y = \frac{607034035.4099546391}{0.1074037156+(0.7809442825)^x}$
1981 , 1991 , 2001	$y = \frac{930490540.3659358186}{0.36170191551+(0.7377753206)^x}$
1991 , 2001 , 2011	$y = \frac{1400316634.9560702305}{0.6546286037+(0.7088532327)^x}$
1951 , 1981 , 2011	$y = \frac{392165083.5070385244}{0.0860648533+(0.4878388268)^x}$

Table 7:

Year	x_i	y_i	$\log y_i$	$\Delta \log y_i$	$\Delta^2 \log y_i$
1951	0	361088090	19.704632503150	0.195912110819	0.02562041346
1961	1	439234771	19.900544613969	0.221532524279	−0.00112252063
1971	2	548159652	20.122077138248	0.220410003649	
1981	3	683329097	20.342487141897		

Table 8: Makeham's curve representing total population of India

Years $x = (0, 1, 2, 3)$	Curve Representing total population
1951,1961,1971,1981	$y = 352696247.934125408029 \times (1.246646468467)^x \times (1.023793397619)^{-0.043813525170^x}$
1961, 1971, 1981, 1991	$y = 439256180.12431468776914 \times (1.24827978453195169)^x \times (0.999951259317406)^{5.799006765234579297866^x}$
1971, 1981, 1991, 2001	$y = 548947760.20731398904366 \times (1.25040580623912214)^x \times (0.9985643293142933046)^{3.128582992622534292060511^x}$
1981, 1991, 2001, 2011	$y = 757632444.355018001412 \times (1.29660574080506142197)^x \times (0.9019269199614684234)^{1.444183208608258161559716^x}$
1951, 1971, 1991, 2011	$y = 360890060.48806901244 \times (1.52270198186421084696 7)^x \times (1.00054872531447159209)^{-4.544806655016345495885^x}$

Thus the Makeham's curve satisfying the data is

$$y = (1.246646468467)^x (1.023793397619)^{-0.043813525170^x}$$

Makeham's curves to the other years obtained have been shown in the following Table 8.

4 Conclusion

The methods described above can be used to represent a given set of numerical data on a pair of variables by non-polynomial curves namely exponential curve , modified exponential curve, logistic curve and Makeham's curve respectively. Each of the four curves, studied here, that represents a given set of numerical data can be used for interpolation at any position of the independent variable lying within its two extreme values. They can also be suitably applied in inverse interpolation also. A set of numerical data on a pair of variables can be represented by each of the four curves namely exponential curve, modified exponential curve, logistic curve and Makeham's curve, applying the corresponding methods described above, only when the values of the independent variable are equally spaced. In the situation where the values of the independent variable are not equally spaced, the methods composed here fail to do so.

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