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AGHM as A Tool of Evaluating the Parameter from Observed Data Containing Itself and Random Error

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Abstract

A number of methods like analytical method, stable mid-range method, and shortest interval method had been developed for determining the value of the parameter from observed data containing the parameter itself and random error. Due to (i) huge computational tasks and (ii) limitation of finite set of observed data in determining the appropriate value of the parameter involved in these methods, three more methods have recently been developed for the same purpose. These three methods are respectively based on Arithmetic-Geometric Mean (abbreviated as AGM), Arithmetic-Harmonic Mean (abbreviated as AHM), and Geometric-Harmonic Mean (abbreviated as GHM). Due to the variation occured in accuracy of values of the parameter yielded by these three methods, one more method has been developed in this study for determining the value of the said parameter with an objective of finding more accurate value of the parameter. The method is based on Arithmetic-Geometric-Harmonic Mean (abbreviated as AGHM). This paper describes the derivation of the method and one numerical application of the method in determining the central tendency, which can be represented by the said parameter, of sex ratio in the populations of the different states of India.

Keywords: AGHM, observed data, parameter, random error, Sex ratio,; central tendency

1. Introduction

In the case of experimental research or survey research, sometimes the observed numerical data

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

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are found to be composed of some parameter μ and random errors ε_i.

In this situation, the numerical data can be expressed as

$$x_i = \mu + \varepsilon_i \ , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , N) \qquad \text{----------------------} \qquad (1.1)$$

[Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*a* , 2015*b* , 2015*c* , 2015*d* , 2015*e* , 2015*f* , 2015*g* , 2017*b*)],. The existing statistical methods of estimation namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [Aldrich (2000.**),** Anders (1999) , Barnard (1949), Birnbaum (1962), Ivory (1825), Kendall & Stuart (1977), Lehmann & Casella George (1998), Lucien (1990) , Walker & Lev (1965)] provide $\frac{1}{N} \sum_{i=1}^{N} x_i$ as estimator of the parameter μ which suffers from an error

$$\frac{1}{N} \sum_{i=1}^{N} \varepsilon_i$$

[Chakrabarty (2014*a* , 2014*b* , 2014*c*)].

A number of attempts had been made on developing method(s) of determining the appropriate value of the parameter μ involved in the model described by equation (1.1) [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*a* , 2015*b* , 2015*c* , 2015*d* , 2015*e* , 2015*f* , 2015*g* , 2016*a* , 2016*b* , 2019*a* , 2019*c*]. In these studies some methods have been developed for determining the appropriate value of the parameter μ when ε_i occurs due to random cause.

The first method, developed for the same is based on computing sequence of interval value of μ with decreasing length of interval and then to find out the shortest interval value of μ [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*d*)] while the second one is based on stable mid range and median (Chakrabarty , 2015*b*) and the third one on the convergence of statistic i.e. some function of the available numerical data (Chakrabarty , 2017*a*). The fourth one (Chakrabarty , 2018*a*) has been developed on the basis of Pythagorean means [Kolmogorov (1930) , O'Meara (1989) , Riedweg (2005) , Cornelli, McKirahan & Macris (2013) , *de Carvalho (2016)* , Chakrabarty (2018*b* , 2018*c* , 2018*d* , 2018*e* , 2018*f* , 2019*d* , 2019*e* , 2019*f*], 2020*a* , 2021*b*)] while the fifth one [Chakrabarty (2016*c* , 2019*b*)] for the same is based on the probabilistic convergence of Pythagorean means [Chakrabarty (2017*b* , 2017*b*)}.

The methods, developed so far, for determining the appropriate value of the parameter from observed data containing the parameter itself and random error involve huge computational tasks. Moreover, a finite set of observed data may not yield the appropriate value of the parameter in many situations while the number of observations required in the methods may be too large for obtaining the appropriate value of the parameter. However, the appropriate value of theparameter is not perfectly attainable in practical situation. What one can expect is to obtain that value which is more and more close to the appropriate value of the parameter. In order to obtain such value of parameter, three methods have already been developed which involves lesser computational tasks than those involved in the earlier methods as well as which can be applicable in the case of finite set of data [Chakrabarty (2019*g* , 2020*b* , 2020*c* , 2020*d*)]. The methods developed are based on the concepst of Arithmetic-Geometric Mean (abbreviated as *AGM*) [Chakrabarty (2019*g* , 2020*b* , 2021*a* , 2021*e*)], Arithmetic-Harmonic Mean(abbreviated as *AHM*) (Chakrabarty , 2020*c* , 2021*a* , 2021*c* , 2021*d* , 2021*e*) and Geometric-Harmonic Mean (abbreviated as *GHM*) [Chakrabarty , 2020*d* , 2021*a* , 2021*e*)] respectively. However, It has been found that

$$AGM > AHM > GHM$$

This means, the accuracy of value of the parameter yielded by *AGM* , *AHM* & *GHM* is different. Moreover, none of them may yield reasonably accurate value of the parameter. Accordingly, one more method has been developed for determining the value of the said parameter with an objective of finding reasonably accurate value of the parameter. The method is based on Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) [Chakrabarty (2020*e*, 2021*a* , 2021*e*]. This paper describes the derivation of the method and one numerical application of the method in determining the central tendency, which can be represented by the said parameter, of sex ratio in the populations of the states in India.

2. Arithmetic-Geometric-Harmonic Mean (*AGHM*)

Let a_0 , g_0 & h_0 be respectively the *AM* , the *GM* & the *HM* of n numbers (or values or observations)

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

i.e.

$$AM(x_1 , x_2 , \ldots\ldots\ldots , x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i = a_0 ,$$

$$GM(x_1 , x_2 , \ldots\ldots\ldots , x_N) = (\prod_{i=1}^{N} x_i)^{1/N} = g_0$$

$$\& \ HM(x_1, x_2, \ldots\ldots\ldots, x_N) = \left(\frac{1}{N}\sum_{i=1}^{N} x_i^{-1}\right)^{-1} = h_0$$

Then, $h_0 \le g_0 \le a_0$

Let us define the three sequences $\{a'''_n\}$, $\{g'''_n\}$ & $\{h'''_n\}$ respectively by

$$a'''_n = 1/3\,(a'''_{n-1} + g'''_{n-1} + h'''_{n-1}) \quad , \qquad) \text{ ---------------------- } (2.1)$$

$$g'''_n = (a'''_{n-1}\, g'''_{n-1}\, h'''_{n-1})^{1/3} \quad) \text{ ---------------------- } (2.2)$$

$$\& \ h'''_n = \{1/3\,(a'''^{\,-1}_{n-1} + g'''^{\,-1}_{n-1} + h'''^{\,-1}_{n-1})\}^{-1} \quad) \text{ ---------------------- } (2.3)$$

where the square cube takes the principal value.

For $n = 1$, we have

$$h'''_1 \le g'''_1 \le a'''_1$$

Since a'''_1, g'''_1 & h'''_1 are respectively the *AM* , the *GM* & the *HM* of

$$a_0 , \ g_0 \ \& \ h_0$$

therefore, each of a'''_1, g'''_1 & h'''_1 lies between the maximum a_0 and the minimum h_0 of a_0 , g_0 & h_0 ..

Therefore,

$$h_0 \le h'''_1 \le g'''_1 \le a'''_1 \le a_0$$

By the similar logic, we have for $n = 2$ that

$$h_0 \le h'''_1 \le h'''_2 \le g'''_2 \le a'''_2 \le a'''_1 \le a_0$$

Proceeding with the same logic, one can obtain at the n^{th} step that

$$h_0 \le h'''_1 \le h'''_2 \le \ldots\ldots\ldots \le h'''_n \le h'''_{n+1} \le g'''_{n+1} \le a'''_{n+1} \le a'''_n \le \ldots..$$
$$\ldots\ldots. \le a'''_2 \le a'''_1 \le a_0$$

This inequality implies that the values of a'''_n, g'''_n & h'''_n have been increasing starting from h_0 and have been decreasing starting from a_0 .

This means that the values of a'''_n , g'''_n & h'''_n will be more and more close as n becomes more and more large.

Thus, there exists a finite real number M_{AGH} such that

$$\{a'''_n\}\ ,\ \{g'''_n\}\ \&\ \{h'''_n\}$$ converges to M_{AGH} as n approaches infinity.

This common converging point (value) M_{AGH} can be termed as Arithmetic-Geometric-Harmonic Mean of $x_1, x_2, \ldots\ldots\ldots, x_N$.

Accordingly, Arithmetic-Geometric-Harmonic Mean can be defined as follows:

Definition (2.1):

Let a_0, g_0 & h_0 be respectively AM , GM & HM of the n numbers (or values or observations)

$$x_1, x_2, \ldots\ldots\ldots, x_N$$

Then, the three sequences $\{a'''_n\}$, $\{g'''_n\}$ & $\{h'''_n\}$ defined by

$$a'''_{n+1} = 1/3\,(a'''_n + g'''_n + h'''_n)\ ,\quad) \qquad \text{---------------------- (2.4)}$$

$$g'''_{n+1} = (a'''_n\, g'''_n\, h'''_n)^{1/3} \quad) \qquad \text{---------------------- (2.5)}$$

$$\&\ h'''_{n+1} = \{1/3\,(a'''^{\,-1}_n + g'''^{\,-1}_n + h'''^{\,-1}_n)\}^{-1} \quad) \qquad \text{---------------------- (2.6)}$$

respectively converge to a common limit (say, M_{AGH}) which is can be termed as the Arithmetic-Geometric-Harmonic Mean (abbreviated by *AGHM* of $x_1, x_2, \ldots\ldots\ldots, x_N$

and is denoted here by $AGHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$

i.e. $AGHM\,(x_1, x_2, \ldots\ldots\ldots, x_N) = M_{AGH}$

3. E AGHM as a Tool of Evaluation of μ

If the observations

$$x_1, x_2, \ldots\ldots\ldots, x_N$$

are composed of some parameter μ and random errors then the observations can be expressed as

$$x_i = \mu + \varepsilon_i\ ,\quad (i = 1, 2, \ldots\ldots\ldots, N) \quad) \qquad \text{---------------------- (3.1)}$$

where

$$\varepsilon_1, \varepsilon_2, \ldots\ldots\ldots, \varepsilon_N$$

are the random errors associated to

$$x_1, x_2, \ldots\ldots\ldots, x_N$$

Respectively, which assume positive real values and negative real values in random order.

In this case,

$$AM(x_1\, , x_2\, ,\ \ldots\ldots\ldots\, , x_N)\ \longrightarrow \mu\ \text{ as }\ N \longrightarrow \infty$$

On the other hand, if the observations

$$x_1\, , x_2\, ,\ \ldots\ldots\ldots\, , x_N$$

are composed of some parameter μ and random errors then the observations can also be expressed as

$$x_i = \mu\ \varepsilon_i^{\prime}\ \ ,\quad (i = 1\, , 2\, , \ldots\ldots\ldots\ldots\, , N)$$

where

$$\varepsilon_1^{\prime}\, , \varepsilon_2^{\prime}\, , \ldots\ldots\ldots\ldots\ldots\, , \varepsilon_N^{\prime}$$

are the random errors associated to

$$x_1\, , x_2\, ,\ \ldots\ldots\ldots\, , x_N$$

respectively which assume positive real values in $(0\, , 1)$ and in $(1\, , \infty)$ in random order.

In this case,

$$GM(x_1\, , x_2\, ,\ \ldots\ldots\ldots\, , x_N)\ \longrightarrow \mu\ \ \text{ as }\ \ N \longrightarrow \infty$$

Again since the observations

$$x_1\, , x_2\, ,\ \ldots\ldots\ldots\, , x_N$$

consist of μ and random errors,

therefore, the reciprocals

$$x_1^{-1},\ x_2^{-1}\ ,\ \ldots\ldots\ldots\ldots\ ,\ x_N^{-1}$$

are composed of μ^{-1} and random errors different from the respective random errors

$$\varepsilon_1\, , \varepsilon_2\, ,\ \ldots\ldots\ldots\, , \varepsilon_N$$

provided $x_1\, , x_2\, ,\ \ldots\ldots\ldots\, , x_N$ are all different from zero.

In this case thus

$$x_i^{-1} = \mu^{-1}\ +\ \varepsilon_i^{\prime\prime}\quad ,\quad (i = 1\, , 2\, , \ldots\ldots\ldots\ldots\, , N)$$

where

$$\varepsilon_1^{\prime\prime}\, , \varepsilon_2^{\prime\prime}\, , \ldots\ldots\ldots\ldots\ldots\, , \varepsilon_N^{\prime\prime}$$

are the random errors associated to

$$x_1^{-1}\ ,\ x_2^{-1}\ ,\ \ldots\ldots\ldots\ldots\ ,\ x_N^{-1}$$

respectively which assume positive real values and negative real values in random order.

In this case,

$$HM(x_1\ , x_2\ ,\ \ldots\ldots\ldots\ , x_N) \rightarrow \mu \text{ as } N \rightarrow \infty$$

This implies that the common converging value of

$$AM(x_1\ , x_2\ ,\ \ldots\ldots\ldots\ , x_N)\ ,$$

$$GM(x_1\ , x_2\ ,\ \ldots\ldots\ldots\ , x_N)$$

$$\&\ HM(x_1\ , x_2\ ,\ \ldots\ldots\ldots\ , x_N)$$

as $N \rightarrow \infty$,

is the value of μ.

It is to be noted that a finite set of observed values may not be sufficient for obtaining the common converging value.

However, the three sequences $\{a^{///}_n\}$, $\{g^{///}_n\}$ & $\{h^{///}_n\}$ defined by (2.1) , (2.2) & (2.3) respectively converge to a common finite real number as n approaches infinity which is the $AGHM\ (x_1\ , x_2\ ,\ \ldots\ldots\ldots\ , x_N)$.

Now, from the model described by Equation (1.1), it follows that

$$a_0 = \mu + \delta_0\quad ,\quad g_0 = \mu + d_0\quad \&\quad h_0 = \mu + e_0$$

for some real numbers $\delta_0\ , d_0\ , e_0$.

Since $a_0 > g_0 > h_0$

therefore $\delta_0 > d_0 > e_0$

Thus $a^{///}_1 = \mu + \delta_1$ where $\delta_1 = 1/3\ (\delta_0 + d_0 + e_0)$

Here, $\delta_1 < 1/3\ (\delta_0 + \delta_0 + \delta_0)$, since $d_0 < \delta_0$ & $e_0 < \delta_0$

i.e. $\delta_1 < \delta_0$

In general, $a^{///}_{n+1} = \mu + \delta_{n+1}$ where $\delta_{n+1} = 1/3\ (\delta_n + d_n + e_n)$

Now, $\delta_{n+1} = 1/3(\delta_n + d_n + e_n) < 1/3\,(\delta_n + \delta_n + \delta_n)$, since $d_n < \delta_n$ & $e_n < \delta_n$

i.e. $\delta_{n+1} < \delta_n$

This implies that the value of a'''_n becomes closer and closer to μ as n becomes larger and larger.

Thus, the converging point (value) of the sequence $\{a'''_n\}$ becomes closest to μ.

Again the three sequences $\{a'''_n\}$, $\{g'''_n\}$ & $\{h'''_n\}$ converge to the same point (value) as n approaches infinity which is the $AGHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$.

Therefore, the $AGHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ is that value which is closest to μ.

Hence, $AGHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be regarded as a measure of the value of the parameter μ .

Note

The parameter μ can , in this case, be interpreted as the value of the central tendency of $x_1, x_2,$ $\ldots\ldots\ldots, x_N$.

Accordingly, $AGHM\,(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be regarded as a measure of the central tendency of $x_1, x_2, \ldots\ldots\ldots, x_N$.

3.1. Remark (Two Properties of AGHM):

Property (3.1): If $y_i = x_i - a$, for finite real a , $(i = 1, 2, \ldots\ldots\ldots, N)$
then from equation (3.1),

$$y_i = (\mu - a) + \varepsilon_i \;, \quad (i = 1, 2, \ldots\ldots, N)$$

This means that if a finite real number a is subtracted from all the observed values, the value of the parameter is decreased by a.

Similarly if a finite real number a is added from all the observed values, the value of the parameter is increased by a .

Property (3.2): If $y_i = c.x_i$, for non-zero finite real c , $(i = 1, 2, \ldots\ldots\ldots, N)$ then from equation (3.1),

$$y_i = c.\mu + c.\varepsilon_i \quad , \quad (i = 1, 2, \ldots\ldots\ldots, N)$$

where $c.\varepsilon_i$ $(i = 1, 2, \ldots\ldots\ldots, N)$ are also random.

This means that if all the observed values are multiplied by a non-zero finite real number, the value of the parameter is changed by c times.

4. Application to Numerical Data

The following table (**Table – 1)** shows the observed data on the population (sex-wise) of India for different states in 2011, as published in "Census Report" by Register General of India.

Table – 1

Population of India in 2011 (State-wise)

State	Number of Persons	Number of Males	Number of Females
Jammu & Kashmir	1,25,41,302	66,40,662	59,00,640
Himachal Pradesh	68,64,602	34,81,873	33,82,729
Punjab	2,77,43,338	1,46,39,465	1,31,03,873
Chandigarh	10,55,450	5,80,663	4,74,787
Uttarakhand	1,00,86,292	51,37,773	49,48,519
Haryana	2,53,51,462	1,34,94,734	1,18,56,728
Delhi	1,67,87,941	89,87,326	78,00,615
Rajasthan	6,85,48,437	3,55,50,997	3,29,97,440
Uttar Pradesh	19,98,12,341	10,44,80,510	9,53,31,831
Bihar	10,40,99,452	5,42,78,157	4,98,21,295
Sikkim	6,10,577	3,23,070	2,87,507
Arunachal Pradesh	13,83,727	7,13,912	6,69,815
Nagaland	19,78,502	10,24,649	9,53,853
Manipur	28,55,794	14,38,586	14,17,208
Mizoram	10,97,206	5,55,339	5,41,867
Tripura	36,73,917	18,74,376	17,99,541

Meghalaya	29,66,889	14,91,832	14,75,057
Assam	3,12,05,576	1,59,39,443	1,52,66,133
West Bengal	9,12,76,115	4,68,09,027	4,44,67,088
Jharkhand	3,29,88,134	1,69,30,315	1,60,57,819
Odisha	4,19,74,218	2,12,12,136	2,07,62,082
Chhattisgarh	2,55,45,198	1,28,32,895	1,27,12,303
Madhya Pradesh	7,26,26,809	3,76,12,306	3,50,14,503
Gujarat	6,04,39,692	3,14,91,260	2,89,48,432
Daman & Diu	2,43,247	1,50,301	92,946
Dadra & Nagar Haveli	3,43,709	1,93,760	1,49,949
Maharashtra	11,23,74,333	5,82,43,056	5,41,31,277
Andhra Pradesh	8,45,80,777	4,24,42,146	4,21,38,631
Karnataka	6,10,95,297	3,09,66,657	3,01,28,640
Goa	14,58,545	7,39,140	7,19,405
Lakshadweep	64,473	33,123	31,350
Kerala	3,34,06,061	1,60,27,412	1,73,78,649
Tamil Nadu	7,21,47,030	3,61,37,975	3,60,09,055
Pondicherry	12,47,953	6,12,511	6,35,442
Andaman & Nicobar	3,80,581	2,02,871	1,77,710
India	**1,21,08,54,977**	**62,32,70,258**	**58,75,84,719**

Table – 2 has been prepared for observed values on the two ratios **Male/Female & Female/Male**.

Table – 2

State	Value of the Ratio Male/Female	Value of the Ratio Female/Male
Jammu & Kashmir	1.1254138534125111852273651671683	0.88856201384741461016988968870875
Himachal Pradesh	1.0293088804926436613751796256809	0.97152567023553127871119940330966
Punjab	1.11718611741734676457868601138	0.89510600284914783429585712319405
Chandigarh	1.2229968385823537712700642603947	0.81766360177934533455722165869015
Uttarakhand	1.0382445737805593956494862402266	0.96316419584905755859591305415792
Haryana	1.1381499179200197558719403869263	0.878618874592118673847146598073
Delhi	1.1521304409972803426396508480421	0.86795727672502366109786158864161
Rajasthan	1.077386518469311558714857879884	0.92817200035205763708961523638845
Uttar Pradesh	1.0959666766496911194331303675474	0.91243650131493423988837726768373
Bihar	1.0894569681498644304609103396449	0.91788847952225054362107394324387
Sikkim	1.1236943796151050235298618816238	0.88992168879809329247531494722506

Arunachal Pradesh	1.0658345961198241305435082821376	0.9382318829211443427201111621600 4
Nagaland	1.0742210801874083323111632505218	0.9309070715923208825656395507144 4
Manipur	1.0150845888535768920299631387912	0.9851395745544583361717686672885 7
Mizoram	1.0248621894302476437945104610541	0.9757409438199009974087899463210 8
Tripura	1.0415856043291039214999824955364	0.9600747128644412860600007682556 8
Meghalaya	1.0113724418785172369610123540989	0.9887554362689632612787498860461 5
Assam	1.0441048168517855831597956077024	0.9577582478885868220112835812393 2
West Bengal	1.0526667948213744061675457587868	0.9499682187369543058436143096928 7
Jharkhand	1.0543346515488809532602154750904	0.948465459738935749275781342520 8
Odisha	1.0216767277963741786589610810708	0.9787831833625807415151402008736 9
Chhattisgarh	1.0094862433659738915914763831542	0.9906028998133312865101756072967 2
Madhya Pradesh	1.0741921997293521487367677330733	0.9309320997228938847833472374706 3
Gujarat	1.0878399216924771607664276945985	0.919252897470599779113315885105 9
Daman & Diu	1.6170787338884943945947109074086	0.6183990791811099061217157570475 2
Dadra & Nagar Haveli	1.292172672041827554701931990 21	0.7738903798513625103220478943022 3
Maharashtra	1.0759593940486569345112623151307	0.9294031034360559651952328875050 8
Andhra Pradesh	1.0072027731513157131279371653056	0.9928487357825874308994648856822 6
Karnataka	1.0278146308628600560795309711955	0.9729380862777664376235381171432 2
Goa	1.0274323920462048498411882041409	0.9733000514110993857726547068214 4
Lakshadweep	1.0565550239234449760765550239234	0.9464722398333484285843673580291 6
Kerala	0.9222472932159456123430538242644 8	1.084307872038230501593145543397 8
Tamil Nadu	1.0035802105886977594941050244168	0.9964325615920648569821634997533 8
Pondicherry	0.9639133075874745452771456718315 8	1.037437694996498022076338220864 6
Andaman & Nicobar	1.1415846041303246862866467840864	0.8759753735132177590685706680600 2
India	1.0607325851848778252519531570732	0.9427446785050988266473642642514 8

Central Tendency of the Ratio Male/Female:

From the observed values on the ratio **Male/Female** in **Table – 3** it has been obtained that

AM of **Male / Female** = 1.0835068016450523020161865887443 ,

GM of **Male / Female** = 1.0784172361960199316030087704149

& HM of **Male / Female** = 1.0740468088974845410059550737324

The following table (**Table – 3)** shows the values of a'_n & h'_n , in this case, for $n =$ 1 , 2 , 3 , ………..
:

Table – 3

Computeed Values of $a^{///}_n$, $g^{///}_n$ & $h^{///}_n$ of the **Ratio Male / Female**

n	Term of sequence	Values of $a^{///}_n$, $g^{///}_n$ & $h^{///}_n$
1	$a^{///}_1$	1.0786569489128522582083834776305
	$g^{///}_1$	1.0786500232826463705959730067094
	$h^{///}_1$	1.0786430991879167734854095348694
2	$a^{///}_2$	1.0786500237944718007632553397364
	$g^{///}_2$	1.0786500237796527462927936308012
	$h^{///}_2$	1.0786500237648336918293636455239
3	$a^{///}_3$	1.0786500237796527462951375386872
	$g^{///}_3$	1.0786500237796527462950696747305
	$h^{///}_3$	1.0786500237796527462950018107739
4	$a^{///}_4$	1.0786500237796527462950696747305
	$g^{//}_4$	1.0786500237796527462950696747305
	$h^{///}_4$	1.0786500237796527462950696747305

The digits in $a^{///}_n$, $g^{///}_n$ & $h^{///}_n$, which are agreed, have been underlined in the above table.

It is obsereved that the values of $a^{///}_n$, $g^{///}_n$ & $h^{///}_n$ become identical at n = 4 which is

1.0786500237796527462950696747305

Therefore, this value can be regarded as the $AGHM$ and consequently the central tendency of the data on the Ratio **Male/Female** in the context of the states inIndia.

Central Tendency of the Ratio Female/Male:

From the observed values on **Female/Male** in **Table – 3** it has been obtained that

AM of **Female/Male** = 0.9310581175009550726813265197974 ,

GM of **Female/Male** = 0.92728488235905168784178691872109

& *HM* of **Female/Male** = 0.92292913942185992242619179784686

The computed values of $\{a'_n\}$ & $\{h'_n\}$,in this case, have been shown in the following table **Table – 4**:

Table – 4

Computeed Values of a'''_n , g'''_n & h'''_n of the **Ratio Male / Female**

n	Term of sequence	Values of a'''_n , g'''_n & h'''_n
1	a'''_1	0.9270907130939555609831017454551 2
	g'''_1	0.92708476189219240096038081993126
	h'''_1	0.92707880944712926028363970338239
2	a'''_2	0.92708476147775907407570742292292
	g'''_2	0.92708476146502230193490281506759
	h'''_2	0.92708476145228552978840450686974
3	a'''_3	0.92708476146502230193300491495342
	g'''_3	0.92708476146502230193294658682228
	h'''_3	0.9270847614650223019328882586911
4	a'''_4	0.92708476146502230193294658682227
	g''_4	0.92708476146502230193294658682227
	h'''_4	0.92708476146502230193294658682227

The digits in a'''_n , g'''_n & h'''_n, which are agreed, have been underlined in the above table.

It is observed that the values of a'''_n , g'''_n & h'''_n become identical at $n = 4$ which is

0.92708476146502230193294658682227

Therefore, this value can be regarded as the $AGHM$ and consequently the central tendency of the data on the Ratio **Female/Male** in the context of the states inIndia.

5. Discussions and Conclusion:

T The methods developed so far, for determining appropriate value of parameter from observed data containing the parameter itself and random error involve huge computational tasks. The method, based on

AGHM, described here involves lesser computational tasks than those involved in the methods developed so far.

Moreover, a finite set of observed data may not yield the appropriate value of the parameter in many situations while the number of observations required in the methods may be too large for obtaining the appropriate value of the parameter. The method, based on AGHM, can be applicable in the case of finite set of data.

In this connection it is to be mentioned that AGHM exists only when the observed values are strictly positive. In the situation where the numerical values in the data are found not to be strictly positive, the central tendency of the data can be determined by the application of the two properties of AGHM namely Property (3.1) & Property (3.2) for which suitable change of origin and scale is required to be applied in order to convert the numerical values in the data into strictly positive ones.

It is to be noted that AGHM may not be able to yield the actual value of the parameter. However, it can at least yield that value which is very close to the actual value of the parameter.

If μ is the central tendency of

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

then the central tendency of

$$x_1^{-1} , x_2^{-1} , \ldots\ldots\ldots\ldots , x_N^{-1}$$

should logically be $\mu - 1$.

It is observed in the in the above example that the AGHM of the ratio Male /Female is

1.0786500237796527462950696747305

and of the ratio Female/Male is

0.92708476146502230193294658682227

These two values are reciprocals each other i.e.

$(1.0786500237796527462950696747305)^{-1} = 0.92708476146502230193294658682227$

& $(0.92708476146502230193294658682227)^{-1} = 1.0786500237796527462950696747305$

It is to be noted that AM of the ratio Male/Female which is 1.0835068016450523020161865887443 and of the ratio Female/Male which is 0.9310581175009550726813265197974 are not reciprocals each other.

Similarly, HM of the ratio Male/Female which is 1.0740468088974845410059550737324 and of the ratio Female/Male which is 0.92292913942185992242619179784686 are also not reciprocals each other. Thus, AGHM can logically be regarded as a measure of central tendency of data which is more meritorious than AM & HM .

Of course, GM of the ratio Male/Female which is 1.0784172361960199316030087704149 and of the ratio Female/Male which is 0.92728488235905168784178691872109 are reciprocals each other. This implies that GM can also logically be regarded as a measure of central tendency of data which is more meritorious than AM & HM . However, which one of AHM and GM is more meritorious as ameasure of central tendency of data is still unknown.

On the whole, the two values

1.0786500237796527462950696747305 & 0.92708476146502230193294658682227

can logically be regarded as the respective values of central tendency of the Ratio Male/Female and the Ratio Female/Male of the states in India while the overall values of these two ratios in India (combing the states) are

1.0607325851848778252519531570732 & 0.94274467850509882664736426425148

Respectively at this stage.

However, it is yet to be determined the size of errors or discrepancies in values obtained by AGHM. It is also to be assessed the performance of AGHM by applying it in the data with various sample sizes.

It has already been derived that each of AGM, AHM & GHM converges to μ in the model described by the equation (1.) as $n \to \infty$ [Chakrabarty (2020b, 2020c , 2020d)].

Thus, there is necessity of study on the comparison of the accuracy of the findings yielded by AGM, AHM, GHM & AGHM.

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