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Observed Data Containing One Parameter and Chance Error: Probabilistic Evaluation of Parameter by Pythagorean Mean

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Abstract

Recently some methods have been developed for determining the appropriate value of the parameter from observed data containing the parameter itself and chance error since the existing statistical methods of estimation in such situation fail in finding out the appropriate value of the parameter. However, the methods are based on some probabilistic assumptions. Accordingly, the value of the parameter obtained by the methods is not deterministic but probabilistic i.e. one cannot be fully certain that the value of the parameter obtained is identical with its actual value. This paper is based on the evaluation of the probability that the value obtained is the actual value of the parameter and on one numerical application of the method in determining the central tendency of each of annual maximum and annual minimum of ambient air temperature at Guwahati.

Key words*: 1; Observed data 2; parameter 3; chance error 4; probabilistic evaluation of parameter*

1. Introduction:

There are many situations where observed data

$$x_1 , x_2 , \ldots\ldots\ldots\ldots , x_n$$

are composed of some parameter μ and chance errors ε_i i.e.

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n) \qquad \text{----------------------------------} (1.1)$$

[Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*a* , 2015*b* , 2015*c* , 2015*d* , 2015*e*]. The existing methods of estimation of μ namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [Aldrich (1930), Anders (1999) , Barnard (1949), Birnbaum (1962), Ivory (1825) , Kendall & Stuart (1977), Lehmann & Casella George (1998), Lucien (1990), Walker & Lev (1965)]

provides $\bar{X}$ as estimator of the parameter μ where $\bar{X}$ is given by

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \text{----------------------------------- (1.2)}$$

It has been shown that this estimator $\bar{X}$ of the parameter μ suffers from an error $\bar{\varepsilon}$ given by

$$\bar{\varepsilon} = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i \quad \text{----------------------------------- (1.3)}$$

which is not zero usually. In other words, none of these methods can provide appropriate value of the parameter μ [Chakrabarty (2014*a* , 2014*b* , 2014*c*)].

Recently, some studies have been done on determining the true value of the parameter μ involved in the model described by (1.1) [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*a* , 2015*b* , 2015*c* , 2015*d* , 2015*e* , 2015*f* , 2016*a* , 2016*b*) , Bordoloi & Chakrabarty (2015 , 2015 – 16 , 2016*a* , 2016*b* , 2016*c* , 2016 – 17)]. In these studies some methods have been developed for determining the true value of the parameter μ when ε_i occurs due to chance only. One of them is based on computing sequence of interval value of μ with decreasing length of interval and then to find out the shortest interval value of μ [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*d*) , Bordoloi & Chakrabarty (2016*a* , 2016*b*]. The other one is based on stable mid range and median. However, these methods may not be always successful in determining the true value μ [Chakrabarty (2015*b*) , Bordoloi & Chakrabarty (2015)]. For this reason, another method has been derived for determining the true value μ which is based on the convergence of statistic i.e. some function of the available numerical data (Chakrabarty , 2017). However, in some situations, the available data may not be sufficient obtaining the converging point of the statistic considered. One method has been developed for determining the true value μ in such situation (Chakrabarty, 2018*a*). The method is based on the Pythagorean means [Kolmogorov (1930), O'Meara (1989), Riedweg (2005), Cornelli, McKirahan & Macris (2013), de Carvalho (2016), Chakrabarty (2018*b*, 2018*c*, 2018*d*, 2018*e*)]. The methods, developed so far, for determining the appropriate value of the parameter from observed data containing the parameter itself and chance error are based on some probabilistic assumptions. Accordingly, the value of the parameter obtained by the methods is not deterministic but probabilistic i.e. one cannot be fully certain that the value of the parameter obtained is identical with its actual value. This paper is based on the evaluation of the probability that the value obtained is the actual value of the parameter and on one numerical application of the method in determining the central tendency of each of annual maximum and annual minimum of ambient air temperature at Guwahati.

2. Probabilistic Evaluation of μ:

2.1. Probabilistic Convergence of Pythagorean Arithmetic Mean of Observations

If the observations

$$x_1 , x_2 , \ldots\ldots\ldots\ldots , x_n$$

are composed of some parameter μ and chance errors then the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n) \qquad \text{-------------------------------- (2.1)}$$

where

(i) $x_1 , x_2 , \ldots\ldots\ldots\ldots , x_n$ are observed data,

(ii) μ is the parameter

& (iii) $\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots\ldots , \varepsilon_n$ are the chance errors associated to

$$x_1 , x_2 , \ldots\ldots\ldots\ldots , x_n$$

respectively which assume positive and negative values in random order.

From Equations described by (2.1),

$$\sum_{i=1}^{n} x_i = n\mu + \sum_{i=1}^{n} \varepsilon_i$$

which implies

$$A(x_1 , x_2 , \ldots\ldots\ldots\ldots , x_n) = \mu + A(\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots\ldots , \varepsilon_n) \qquad \text{----------------------------- (2.2)}$$

where

$$A(x_1 , x_2 , \ldots\ldots\ldots\ldots , x_n) = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \text{---------------------------------- (2.3)}$$

$$\& \quad A(\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots\ldots , \varepsilon_n) = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i \qquad \text{---------------------------------- (2.4)}$$

Now,

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots\ldots , \varepsilon_n$$

are independently and identically distributed random variables with arithmetic expectation zero (0). Therefore by the law of large numbers, the series

$$\{ A_n(\varepsilon) = A(\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n) = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i \}$$

converges to 0 with probability approaching 1 that is with probability approaching certainity as $n \to \infty$.

Accordingly with probability approaching 1 that is with probability approaching certainity, the series

$$\{ A_n(x) = A(\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n) = \frac{1}{n} \sum_{i=1}^{n} x_i \} \quad \text{---------------------------- (2.5)}$$

converges to μ as $n \to \infty$.

Note (2.1.1): Since

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n$$

are random, each of them can assume either positive value or negative value with equal probability i.e.

$$P(\varepsilon_i < 0) = P(\varepsilon_i > 0) = ½ \quad \text{for all} \quad i = 1 , 2 , \ldots\ldots\ldots , n$$

This implies, the probability that all of them assume positive values is $(½)^n$.

Similarly, the probability that all of them assume negative values is $(½)^n$.

Therefore, the probability that all of them assume values with same sign is $(½)^n + (½)^n = (½)^{n-1}$.

The series $\{A_n(\varepsilon)\}$ will never converge to 0 if all

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n$$

assume values with same sign

and hence the series $\{ A_n(x) \}$ will never converge to μ with probability $(½)^{n-1}$ i.e. the converging point of the series $\{ A_n(x) \}$, if exits, is not the actual value of μ with probability $(½)^{n-1}$.

2.2. Probabilistic Convergence of Pythagorean Geometric Mean of Observations

If the observations

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

are composed of some parameter μ and chance errors then the observations can also be expressed as

$$x_i = \mu \,.\, e_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \quad \text{-------------------------------- (2.6)}$$

where

$$e_1 , e_2 , \ldots\ldots\ldots , e_n$$

are the chance errors associated to

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

respectively assuming positive values either pure decimal fraction or greater than 1 occurred in random order.

From Equations described by (2.6),

$$\left(\prod_{i=1}^{n} x_i \right)^{1/n} = \mu \,.\, \left(\prod_{i=1}^{n} e_i \right)^{1/n}$$

which implies

$$G\,(x_1 , x_2 , \ldots\ldots\ldots , x_n) = \mu \,.\, G\,(e_1 , e_2 , \ldots\ldots\ldots , e_n) \qquad \text{-----------------------------} \ (2.7)$$

where

$$G(x_1 , x_2 , \ldots\ldots\ldots , x_n) = \left(\prod_{i=1}^{n} x_i \right)^{1/n} \qquad \text{-------------------------------} \ (2.8)$$

$$\&\ \ G\,(e_1 , e_2 , \ldots\ldots\ldots , e_n) = \left(\prod_{i=1}^{n} e_i \right)^{1/n} \qquad \text{-----------------------------} \ (2.9)$$

Now,

$$e_1 , e_2 , \ldots\ldots\ldots , e_n$$

are independently and identically distributed random variables with geometric expectation one (1).

Therefore by the similar logic of the law of large numbers, the series

$$\{\ G_n\,(e) = G\,(e_1 , e_2 , \ldots\ldots\ldots , e_n) = \left(\prod_{i=1}^{n} e_i \right)^{1/n} \ \}$$

converges to 1 with probability approaching 1 as $n \rightarrow \infty$.

Accordingly, with probability approaching certainity, the series

$$\{\ G_n\,(x) = G\,(x_1 , x_2 , \ldots\ldots\ldots , x_n) = \left(\prod_{i=1}^{n} x_i \right)^{1/n} \ \} \qquad \text{-----------------------------} \ (2.10)$$

converges to μ as $n \rightarrow \infty$.

Note (2.2.1): Since

$$e_1 , e_2 , \ldots\ldots\ldots , e_n$$

are random, each of them can assume positive values either pure decimal fraction and greater than 1 with equal probability i.e.

$$P\,(0 < e_i < 1) = P\,(e_i > 1) = ½ \quad \text{for all} \quad i = 1 , 2 , \ldots\ldots\ldots , n$$

This implies, the probability that all of them assume values in $0 < e_i < 1$ is $(½)^n$.

Similarly, the probability that all of them assume values in $e_i > 1$ is $(½)^n$.

Therefore, the probability that all of them assume values of same type is $(½)^n + (½)^n = (½)^{n-1}$.

Now, the series $\{G_n(e)\}$ will never converge to 1 if all

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n$$

assume values of same type

and hence the series $\{G_n(e)\}$will never converge to μ with probability $(½)^{n-1}$ i.e. the converging point of the series $\{G_n(e)\}$, if exits, is not the actual value of μ with probability $(½)^{n-1}$.

2.3. Evaluation of μ

Each of the the two series given by (2.5) & (2.10) converges to μ as $n \to \infty$ with probability approaching 1(i.e. with probability approaching certainity).

Therefore, in order to determine the value of μ, it is required to compute the converging values of the two series

$$\{ A_n(x) = A(\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n) = \frac{1}{n} \sum_{i=1}^{n} x_i \}$$

$$\& \quad \{ G_n(x) = G(x_1 , x_2 , \ldots\ldots\ldots , x_n) = (\prod_{i=1}^{n} x_i)^{1/n} \}$$

The common value of them is the actual value of μ with probability approaching 1(i.e. with probability approaching certainity) and is not the actual value of μ with probability $(½)^{n-1}$ where n is the number of observations.

Note (2.3.1): If the series is found to converge but fail to yield a common converging point for the available data then it is to be understood that the available data are insufficient for obtaining the value of μ.

Note (2.3.2): If the series is found either not to converge or to converge to different points then it is to be understood that the errors involved in the data are not only due to chance but also due to some assignable cause(s). Consequently, the data do not follow the model described by the equation (2.1). Accordingly, the value of μ cannot be determined from the given data in this case.

Note (2.3.3): From (2.6),

$$\log G(x_1, x_2, \ldots\ldots\ldots, x_n) = \frac{1}{n} \sum_{i=1}^{n} \log x_i$$

which yields,

$$G(x_1, x_2, \ldots\ldots\ldots, x_n) = \text{antilog}\left\{ \frac{1}{n} \sum_{i=1}^{n} \log x_i \right\} \quad \text{----------------------------------- (2.11)}$$

This formula can be applied in computing the values of the series given by (2.10) since its computation by

$$G(x_1, x_2, \ldots\ldots\ldots, x_n) = \left(\prod_{i=1}^{n} x_i \right)^{1/n}$$

is too complicated.

3. Application to Numerical Data:

Observed data considered here are the data on each of annual maximum & annual minimum of ambient air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013. The objective here is to evaluate, with probability approaching certainity, the central tendency of each of annual maximum & annual minimum of ambient air temperature at Guwahati

3.1. Annual Maximum of Ambient Air Temperature at Guwahati:

The following table shows the observed data on annual maximum of ambient air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013:

Table–3.1.1

Observed Value on Annual Maximum of Ambient Air Temperature (in Degree Celsius)

TPR No (i)	Observed Value (x_i)	TPR No (i)	Observed Value (X_i)	TPR No (i)	Observed Value (x_i)	TPR No (i)	Observed Value (X_i)
1	37.1	12	35.1	23	37.4	34	38.0
2	36.6	13	35.8	24	39.4	35	36.6
3	36.0	14	36.5	25	36.4	36	38.0
4	35.7	15	36.7	26	38.1	37	37.3
5	39.0	16	37.2	27	36.3	38	37.3
6	36.1	17	36.5	28	39.9	39	38.0

7	39.2	18	38.4	29	37.4	40	37.2
8	39.0	19	37.2	30	37.5	41	37.3
9	35.3	20	36.4	31	36.7	42	37.4
10	36.8	21	36.7	32	35.7	43	38.8
11	38.6	22	36.0	33	37.4		

Here the observed values x_i (i = 1 , 2 , 3 , ... , 43) can be assumed to be composed of a parameter μ (representing the central tendency of annual maximum) and chance errors.

3.2 Evaluation of Value of μ (the central tendency of annual maximum)

The computed values of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ & $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ have been shown in **Table–3.1.2**.

In **Table–3.1.2**, it is found that the values of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ and $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ are approaching 37.2.

Hence, with probability approaching 1, the value of the central tendency of annual maximum of the ambient air temperature at Guwahati can be taken as 37.2 Degree Celsius.

However, 37.2 Degree Celsius cannot be the actual value of the central tendency with probability

$$(½)^{42} = 2.273736754 \times 10^{-13}$$

Table–3.1.2

TPR No (n)	Value of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$	TPR No (n)	Value of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$
1	37.1	23	36.92608695652174
2	36.85	24	37.02916666666667
3	36.56666666666667	25	37.004
4	36.35	26	37.04615384615385
5	36.88	27	37.01851851851852
6	36.75	28	37.12142857142857
7	37.1	29	37.13103448275862

8	37.3375	30	37.1433333333333
9	37.1111111111111	31	37.12903225806452
10	37.08	32	37.084375
11	37.21818181818182	33	37.09393939393939
12	37.04166666666667	34	37.12058823529412
13	36.94615384615385	35	37.10571428571429
14	39.75492307692308	36	37.13055555555556
15	36.9	37	37.13513513513514
16	36.91875	38	37.13947368421053
17	36.89411764705882	39	37.16153846153846
18	36.97777777777778	40	37.1625
19	36.98947368421053	41	37.16585365853659
20	36.96	42	37.17142857142857
21	36.94761904761905	43	37.2093023255814
22	36.90454545454545		

Table–3.1.3

TPR No (n)	Value of $G_n = G(x_1, x_2, \ldots\ldots\ldots, x_n)$	TPR No (n)	Value of $G_n = G(x_1, x_2, \ldots\ldots\ldots, x_n)$
1	37.100000000000000000000000000001	23	36.907951375530228273997704899203
2	36.849151957677397816492457471162	24	37.008568334146486997509657949442
3	36.563898828796401481877921649392	25	36.984031371155694976981399533444
4	36.345983740858013867930065117099	26	37.026342595226743557691601271825
5	36.861929466436019045199560948853	27	36.999183617772609404713638815137
6	36.733833524906529592733500969237	28	37.099058031246057185229984821213
7	37.076408237779582297449661223967	29	37.109394916981577631613488308734
8	37.311570055521037921393145135076	30	37.122349300390835138945562995519
9	37.082517567449318065559488651693	31	37.108649562396908895644952409924
10	37.054168483041978324085190136495	32	37.063799115887290185156645453858

11	37.19210244371567988152853618 2712	33	37.07394248015061213002629303 9573
12	37.01309725264882494721802335 3606	34	37.10085461802179646288605030 756
13	36.91834013078424194157538301 5056	35	37.08644981294191541567482780 45
14	36.88830033026850704914049488 2929	36	37.11152717312153438008986658 6791
15	36.87571697219108505830745258 5677	37	37.11660849005270988518502358 9966
16	36.89590158618993756829400378 4643	38	37.12142301102113447511758256 8853
17	36.87249483971790057281815769 6334	39	37.14369484986219186885195992 5933
18	36.95573938654495447431443764 1907	40	37.14510143943119654461481720 7745
19	36.96855513067342480446273092 8845	41	37.14887178893742908890400296 3936
20	36.93991760307707515991880882 1998	42	37.15483139067232867843590968 4177
21	36.92845747149522787157325262 1182	43	37.19228714857607678192581274 7586
22	36.88573999139420977963689188 2147		

3.3. Annual Minimum of Ambient Air Temperature at Guwahati:

The following table (**Table–3.2.1**) shows the observed data on annual maximum of ambient air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from1969 to 2013.

As earlier, the observed values

$$x_i \ (i = 1 , 2 , 3 , \ldots\ldots\ldots\ldots , 43)$$

can in this case also be assumed to be composed of a parameter μ (representing the central tendency of annual minimum) and chance errors.

Table–3.2.1

Observed Value on Annual Minimum of Ambient Air Temperature (in Degree Celsius)

TPR No (i)	Observed Value (x_i)	TPR No (i)	Observed Value (X_i)	TPR No (i)	Observed Value (x_i)	TPR No (i)	Observed Value (X_i)
1	6.6	12	7.5	23	5.9	34	8.0
2	5.9	13	8.3	24	8.4	35	7.9
3	8.2	14	4.9	25	7.8	36	6.7
4	5.0	15	6.1	26	7.5	37	9.6

5	6.3	16	7.8	27	9.4	38	6.4
6	7.4	17	8.6	28	NA	39	7.8
7	6.6	18	7.7	29	NA	40	9.9
8	6.2	19	9.2	30	NA	41	8.6
9	7.3	20	6.7	31	8.3	42	7.0
10	6.2	21	8.6	32	8.9	43	6.4
11	6.4	22	7.4	33	8.6	44	5.6

3.4. Determination of Value of μ (the central tendency of annual minimum)

The computed values of

$$A_n = A(x_1, x_2, \ldots\ldots\ldots, x_n) \text{ and } G_n = G(x_1, x_2, \ldots\ldots\ldots, x_n)$$

have been shown in **Table–3.2.2**.

In **Table–3.2.2**, it is found that the values of

$$A_n = A(x_1, x_2, \ldots\ldots\ldots, x_n) \text{ and } G_n = G(x_1, x_2, \ldots\ldots\ldots, x_n)$$

are not approaching a common value.

Thus, either the data are insufficient to yield the true value of the central tendency of annual minimum of the ambient air temperature at Guwahati or the data do not follow the model described by equation (2.1).

Table–3.2.2

TPR No (n)	Values of $A_n = A(x_1, x_2, \ldots\ldots\ldots, x_n)$	TPR No (n)	Values of $A_n = A(x_1, x_2, \ldots\ldots\ldots, x_n)$
1	6.6	23	7.021739130434783
2	6.6	24	6.975
3	6.366666666666667	25	7.032
4	6.825	26	7.061538461538462
5	6.46	27	7.077777777777778
6	6.433333333333333	28	7.160714285714286
7	6.571428571428571	29	7.220689655172414
8	6.575	30	7.266666666666667

9	6.533333333333333	31	7.290322580645161
10	6.61	32	7.309375
11	6.572727272727273	33	7.290909090909091
12	6.558333333333333	34	7.358823529411765
13	6.630769230769231	35	7.331428571428571
14	6.75	36	7.344444444444444
15	6.626666666666667	37	7.413513513513514
16	6.59375	38	7.444736842105263
17	6.664705882352941	39	7.433333333333333
18	6.772222222222222	40	7.4075
19	6.821052631578947	41	7.363414634146341
20	6.94	42	7.021739130434783
21	6.928571428571429	43	6.975
22	7.004545454545455	44	7.032

Table–3.2.3

TPR No (n)	Values of $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$	TPR No (n)	Values of $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$
1	6.6000000000000000000000000000002	23	6.8994798638029492332170025993497
2	6.2401923047290778184540741626669	24	6.9562840436945652525470112265315
3	6.8349697977658553153553991473341	25	6.9882108302873798619833480810597
4	6.3211333313459467749216627042232	26	7.0072334421441164904282182533205
5	6.3169010013367452786283625052215	27	7.0838901531221111364268314501887
6	6.4857272185701169855041242093667	28	7.1240865180443321452583935939618
7	6.5019299609413096599434266552134	29	7.1789729488032751962564480429371
8	6.4633989337912701904703289281854	30	7.2223221258860362445338145542488
9	6.5514058846887191769812286015146	31	7.2461869803895066094818774956424
10	6.5153871170191872914459649178302	32	7.2657752520059477722135734563005
11	6.5048119729138514238168851259175	33	7.247948125913456784586179510479
12	6.5824407269348502376566800461569	34	7.3081081108055937176215071779575
13	6.7008890789412278622783766593454	35	7.2804552217016962246905047234237

14	6.5527362805560146596136072863623	36	7.2944086947827372183189849879502
15	6.5215350514733093207287599447551	37	7.3548715998389568402950960158236
16	6.5949101184036978619969386660223	38	7.8154612929430738975137961946202
17	6.6987014182595463410514031559311	39	7.7934100068228333088588686838578
18	6.7507455267102729305100790423158	40	7.7551256022354259620074788758758
19	6.8616303970395831831869394230887	41	7.6937848209202825772540315655963
20	6.8534570444635167207666214239384	42	6.8994798638029492332170025993497
21	6.9279444662370421736229802749321	43	6.9562840436945652525470112265315
22	6.9487332418478471976809533313006	44	6.9882108302873798619833480810597

Note: Data corresponding to TPR 29, 30 &31 are not available. Therefore, the values of

$$A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n) \text{ and } G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$$

could not be computed corresponding these three TPR.

4. Conclusion:

The method, developed here, can provide the value of the parameter if the data follow the model described by equation (2.1) and if the data size is sufficiently large for obtaining the common converging point of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ and $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$. Conversely, if the common converging point of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ and $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ is not achieved from the set of data then it implies that either the data do not follow the model described by equation (2.1) or the data size is not sufficient to yield the common converging point.

Regarding the findings obtained on annual maximum and annual minimum of ambient air temperature at Guwahati, the following conclusion can be drawn:

4.1. The central tendency of Annual Maximum of Ambient Air Temperature at Guwahati can be taken as 37.2 Degree Celsius, with probability approaching 1 (i.e. with probability certainity), since all the methods applied have yielded the same numerical results and thus the corresponding data can be treated to follow the model described by equation (2.1).

4.2. The central tendency of Annual Minimum of Ambient Air Temperature at Guwahati is not determinable since the methods applied have yielded different numerical results and thus the corresponding data cannot be treated to follow the model described by equation (2.1).

In this connection, it is to be mentioned that the central tendency of each of annual maximum and annual minimum of the ambient air temperature at Guwahati has been evaluated by another method already developed (Chakrabarty , 2017). The findings obtained by that method have been found to be identical with the findings obtained by the method developed here.

References:

Anders Hald, On the History of Maximum Likelihood in Relation to Inverse Probability and Least Squares, *Statistical Science*, Vol. 14, pp. 214 – 222, 1999.

Aldrich John, Fisher's Inverse Probability of 1930, *International Statistical Review*, Vol. 68, pp. 155 – 172, 2000.

Barnard G. A., Statistical Inference, *Journal of the Royal Statistical Society, Series B*, Vol. 11, pp. 115 – 149, 1949.

Bordoloi Rinamani Sarmah & Chakrabarty Dhritikesh, Central Tendency of Annual Extremum of Ambient Air Temperature at Tezpur Based on Midrange and Median, *J. Chem. Bio. Phy. Sci*. (E- ISSN : 2249 – 1929), Sec. C, 5(4), 4397 – 4410, 2015.

Bordoloi Rinamani Sarmah & Chakrabarty Dhritikesh, Annual Extremum of Ambient Air Temperature at Dibrugarh: Determination of Central Tendency, *J. Chem. Bio. Phy. Sci.* (E- ISSN : 2249 – 1929), Sec. D, 6(1), 212 – 233, 2015 – 16.

Bordoloi Rinamani Sarmah & Chakrabarty Dhritikesh, Determination of Parameter from Observation Containing Itself and Chance Error: Central Tendency of Ambient Air Temperature at Tezpur, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 3(1), 1202 – 1213, 2016*a*.

Bordoloi Rinamani Sarmah & Chakrabarty Dhritikesh, Determination of Parameter from Observation Containing Itself and Chance Error: Central Tendency of Ambient Air Temperature at Dibrugarh, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 3(8), 2488 – 2500, 2016*b*.

Bordoloi Rinamani Sarmah and Chakrabarty Dhritikesh, Confidence Interval of Annual Extremum of Ambient Air Temperature at Guwahati, *Journal of Mathematics and Systems Sciences* (ISSN : 0975 – 5454), 12(1–2), 55 – 62, 2016*c*.

Bordoloi Rinamani Sarmah & Chakrabarty Dhritikesh, Determination of Parameter from Observation Containing itself and Chance Error: Central Tendency of Annual Extremum of Ambient Air Temperature at Dhubri, *J. Chem. Bio. Phy. Sci.* (E- ISSN : 2249 – 1929), Sec. C, 7(1), 062 – 070, 2016 – 17.

Birnbaum Allan, On the Foundations of Statistical Inference, *Journal of the American Statistical Association*, Vol. 57, pp. 269 – 306, 1962.

Chakrabarty Dhritikesh, Determination of Parameter from Observations Composed of Itself and Errors, *International Journal of Engineering Science and Innovative Technology*, 3(2), (ISSN : 2139 – 5967), 304 – 311, 2014*a*.
Chakrabarty Dhritikesh, Analysis of Errors Associated to Observations of Measurement Type, *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 1(1), 15 – 28, 2014*b*.

Chakrabarty Dhritikesh, Observation Composed of a Parameter and Chance Error: An Analytical Method of Determining the Parameter, *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 1(2), 20 – 38, 2014*c*.

Chakrabarty Dhritikesh, Observation Consisting of Parameter and Error: Determination of Parameter, Proceedings of the World Congress on Engineering 2015, (WCE 2015, July 1 - 3, 2015, London, U.K.), ISBN: 978-988-14047-0-1, ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online), Vol. II, 680 – 684, 2015*a*.

Chakrabarty Dhritikesh, Central Tendency of Annual Extremum of Ambient Air Temperature at Guwahati, *J. Chem. Bio. Phy. Sci.* (E- ISSN : 2249 – 1929), Sec. C, 5(3), 2863 – 2877, 2015*b*.

Chakrabarty Dhritikesh, Central Tendency of Annual Extremum of Ambient Air Temperature at Guwahati Based on Midrange and Median, *J. Chem. Bio. Phy. Sci.* (E- ISSN : 2249 – 1929), Sec. D, 5(3), 3193 – 3204, 2015*c*.

Chakrabarty Dhritikesh, Observation Composed of a Parameter and Chance Error: Determining the Parameter as Stable Range, *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2(1), 35 – 47, 2015*d*.

Chakrabarty Dhritikesh, A Method of Finding True Value of Parameter from Observation Containing Itself and Chance Error, *Indian Journal of Scientific Research and Technology*, (E-ISSN: 2321-9262), 3(4), 14 – 21, 2015*e*.

Chakrabarty Dhritikesh, Theoretical Model Modified For Observed Data: Error Estimation Associated To Parameter, *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2(2), 29 – 45, 2015*f*.

Chakrabarty Dhritikesh, Impact of Error Contained in Observed Data on Theoretical Model: Study of Some Important Situations, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 3(1), 1255 – 1265, 2016*a*.

Chakrabarty Dhritikesh, Theoretical Model and Model Satisfied by Observed Data: One Pair of Related Variables, *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 3(2), 1527 – 1534, 2016*b*.

Chakrabarty Dhritikesh, Numerical Data Containing One Parameter and Chance Error: Evaluation of the Parameter by Convergence of Statistic, *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 4(2), 59 – 73, 2017.

Chakrabarty Dhritikesh, Observed Data Containing One Parameter and Chance Error: Evaluation of the Parameter Applying Pythagorean Mean, *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 5(1), 32 – 45, 2018*a*. Online (*http://eses.net.in/ESES Journal).*

Chakrabarty Dhritikesh, Derivation of Some Formulations of Average from One Technique of Construction of Mean, *American Journal of Mathematical and Computational Sciences*, 3(3), 62 – 68, 2018*b*.

Chakrabarty Dhritikesh, One Generalized Definition of Average: Derivation of Formulations of Various Means, *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 7(3), 212 – 225, 2018*c*.

Chakrabarty Dhritikesh, f_H - Mean: One Generalized Definition of Average, *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 7(4), 301 – 314, 2018*d*. Also available in www.jecet.org.

Chakrabarty Dhritikesh, Generalized f_G - Mean: Derivation of Various Formulations of Average, *American Journal of Computation, Communication and Control*, 5(3), 101 – 108, 2018*e*. Available in http://www.aascit.org/journal/ajmcs .

Cornelli G., McKirahan R. & Macris C, *On Pythagoreanism*, Berlin, Walter de Gruyter, 2013.

de Carvalho Miguel, Mean, what do you Mean?, *The American Statistician*, 70, 764 –776, 2016.

George Marsagilia, Evaluating the Normal Distribution, *Journal of Statistical Software*, 11(4), 2004.

Ivory, On the Method of Least Squares, *Phil. M*ag., LXV, 3 – 10, 1825.

Kendall M. G. and Stuart A, Advanced Theory of Statistics, *Vol. 1 & 2, 4th Edition, New York, Hafner Press*, 1977.

Kolmogorov Andrey, On the Notion of Mean, *Mathematics and Mechanics*, 199(1), 144 – 146, 1930.

Lehmann Erich L. & Casella George (1998): *Theory of Point Estimation, 2nd ed. Springer.* ISBN 0 – 387 – 98502 – 6.

Lucien Le Cam (1990): Maximum likelihood — An introduction, *ISI Review*, 58 (2), 153 –171.

Michiel ed. Hazewinkel (2001): Normal Distribution, *Encyclopedia of Mathematics*, Springer, ISBN 978 – 1 – 55608 – 010 – 4.

Walker Helen M. and Lev J., Statistical Inference, *Oxford & IBH Publishing Company*, 1965.

O'Meara Dominic J., *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity*, ISBN 0-19-823913-0, Clarendon Press, Oxford, 1989.

Riedweg Christoph, Pythagoras: his life, teaching, and influence (translated by Steven Rendall in collaboration with Christoph Riedweg and Andreas Schatzmann, Ithaca), ISBN 0-8014-4240-0, *Cornell University Press*, 2005.

Walker Helen M. and Lev J. (1990): De Moivre on the Law of Normal Probability, In Smith, David Eugene, 1985. *A Source Book in Mathematics*, *Dover*, ISBN 0 – 486 – 64690 – 4, 2, 153 –171.

Wlodzimierz Brye (1995): The Normal Distribution: Characterizations with Applications", *Springer – Verlag*, ISBN 0 – 387 – 97990 – 5.