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Observed Data Containing One Parameter and Chance Error: Evaluation of the Parameter Applying Pythagorean Mean

Dhritikesh Chakrabarty
Department of Statistics, Handique Girls' College, Guwahati, Assam, India,
dhritikesh.c@rediffmail.com , dhritikeshchakrabarty@gmail.com

Abstract

In order to evaluate the appropriate value of the parameter from observed data containing the parameter itself and chance error, one method has been developed for evaluation of the parameter as the existing statistical methods of estimation in such situation fail in finding out the appropriate value of the parameter. The method developed here is based on the Pythagorean means This paper describes the derivation of this method with one numerical application in evaluating the central tendency of the observed data on each of annual maximum and annual minimum of ambient air temperature at Guwahati.

Key words*: 1; Observed data 2; parameter 3; chance error 4; Pythagorean mean 5; evaluation of parameter*

1. Introduction:

There are many situations where observed data

$$x_1 , x_2 , \ldots\ldots\ldots\ldots , x_n$$

are composed of some parameter μ and chance errors ε_i i.e.

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n) \qquad \text{---------------------------------- (1.1)}$$

[Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*a* , 2015*b* , 2015*c* , 2015*d* , 2015*e*]. The existing methods of estimation of μ namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [Aldrich (1930), Anders (1999) ,

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Barnard (1949), Birnbaum (1962), Ivory (1825) , Kendall & Stuart (1977), Lehmann & Casella George (1998), Lucien (1990), Walker & Lev (1965)]

provides $\bar{X}$ as estimator of the parameter μ where $\bar{X}$ is given by

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \text{------------------------------------- (1.2)}$$

It has been shown that this estimator $\bar{X}$ of the parameter μ suffers from an error $\bar{\varepsilon}$ given by

$$\bar{\varepsilon} = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i \qquad \text{------------------------------------- (1.3)}$$

which is not zero usually. In other words, none of these methods can provide appropriate value of the parameter μ [Chakrabarty (2014*a* , 2014*b* , 2014*c*)].

Recently, some studies have been done on determining the true value of the parameter μ involved in the model described by (1.1) [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*a* , 2015*b* , 2015*c* , 2015*d* , 2015*e* , 2015*f* , 2016*a* , 2016*b*) , Bordoloi & Chakrabarty (2015 , 2015 – 16 , 2016*a* , 2016*b* , 2016*c* , 2016 – 17)]. In these studies some methods have been developed for determining the true value of the parameter μ when ε_i occurs due to chance only. One of them is based on computing sequence of interval value of μ with decreasing length of interval and then to find out the shortest interval value of μ [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*d*) , Bordoloi & Chakrabarty (2016*a* , 2016*b*]. The other one is based on stable mid range and median. However, these methods may not be always successful in determining the true value μ [Chakrabarty (2015*b*) , Bordoloi & Chakrabarty (2015)]. For this reason, another method has been derived for determining the true value μ which is based on the convergence of statistic i.e. some function of the available numerical data (Chakrabarty , 2017). However, in some situations, the available data may not be sufficient obtaining the converging point of the statistic considered. In order to determine the true value μ in such situation, one method has been introduced here. The method is based on the Pythagorean means [Kolmogorov (1930) , O'Meara (1989) , Riedweg (2005) , Cornelli, McKirahan & Macris (2013) , de Carvalho (2016) , Chakrabarty (2018*b* , 2018*b*)]. This paper describes the derivation of this method and on one numerical application of the method in determining the central tendency of each of annual maximum and annual minimum of ambient air temperature at Guwahati.

2. Method of Determination of Parameter μ:

If the observations

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

are composed of some parameter μ and chance errors then the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad \text{------------------------------------ (2.1)}$$

where

(i) $x_1 , x_2 , \ldots\ldots\ldots , x_n$ are observed data,

(ii) μ is the parameter

& (iii) $\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n$ are the chance errors associated to

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

respectively which assume positive and negative values in random order.

From Equations described by (2.1),

$$\sum_{i=1}^{n} x_i = n\mu + \sum_{i=1}^{n} \varepsilon_i$$

which implies

$$A(x_1 , x_2 , \ldots\ldots\ldots , x_n) = \mu + A(\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n) \text{ ------------------------------------ (2.2)}$$

where

$$A(x_1 , x_2 , \ldots\ldots\ldots , x_n) = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \text{------------------------------------ (2.3)}$$

$$\& \quad A(\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n) = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i \qquad \text{------------------------------------ (2.4)}$$

Now,

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_n$$

being chance errors (i.e. random errors), assume positive and negative values in random order.

Hence, the series $\{ U_n \}$

where $U_n = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i$

approaches 0 as $n \longrightarrow \infty$

Accordingly,

$$A(x_1 , x_2 , \ldots\ldots\ldots , x_n) = \frac{1}{n} \sum_{i=1}^{n} x_i \longrightarrow \mu \quad \text{as} \quad n \longrightarrow \infty \quad \text{------------------------ (2.5)}$$

Again, if the observations

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

are composed of some parameter μ and chance errors then the observations can also be expressed as

$$x_i = \mu \,.\, e_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots , n) \qquad \text{------------------------ (2.6)}$$

where

(i) $x_1 , x_2 , \ldots\ldots\ldots , x_n$ are observed data on the variable X ,

(ii) μ is the parameter

& (iii) $e_1 , e_2 , \ldots\ldots\ldots , e_n$ are the chance errors associated to

$$x_1 , x_2 , \ldots\ldots\ldots , x_n$$

respectively whose magnitude are pure decimal fraction and greater than 1 occurred in random order.

From equations described by (2.1),

$$\left(\prod_{i=1}^{n} x_i \right)^{1/n} = \mu \,.\, \left(\prod_{i=1}^{n} e_i \right)^{1/n}$$

which implies

$$G(x_1 , x_2 , \ldots\ldots\ldots , x_n) = \mu \,.\, G\,(e_1 , e_2 , \ldots\ldots\ldots , e_n) \text{ ------------------------ (2.7)}$$

where

$$G(x_1 , x_2 , \ldots\ldots\ldots , x_n) = \left(\prod_{i=1}^{n} x_i \right)^{1/n} \quad \text{------------------------ (2.8)}$$

$$\& \quad G\,(e_1 , e_2 , \ldots\ldots\ldots , e_n) = \left(\prod_{i=1}^{n} e_i \right)^{1/n} \text{ ------------------------ (2.9)}$$

Since the magnitude of

$$e_1 , e_2 , \ldots\ldots\ldots , e_n$$

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are pure decimal fraction and greater than 1 occurred in random order,

therefore the series $\{ V_n \}$

where $V_n = (\prod_{i=1}^{n} e_i)^{1/n}$

approaches 1 as $n \rightarrow \infty$

Accordingly,

$$G(x_1, x_2, \ldots\ldots\ldots, x_n) = (\prod_{i=1}^{n} x_i)^{1/n} \rightarrow \mu \text{ as } n \rightarrow \infty \quad \text{-----(2.10)}$$

Note:

2.1. From (2.6),

$$\log G(x_1, x_2, \ldots\ldots\ldots, x_n) = \frac{1}{n} \sum_{i=1}^{n} log\ x_i$$

which yields,

$$G(x_1, x_2, \ldots\ldots\ldots, x_n) = \text{antilog} \{ \frac{1}{n} \sum_{i=1}^{n} log\ x_i \} \quad \text{----- (2.11)}$$

This formula can be applied in computing $G(x_1, x_2, \ldots\ldots\ldots, x_n)$ since its computation by

$$G(x_1, x_2, \ldots\ldots\ldots, x_n) = (\prod_{i=1}^{n} x_i)^{1/n}$$

is too complicated.

2.2. In order to determine the value of μ, it is required to compute the converging values of the two series $\{ A_n \}$ & $\{ G_n \}$ where

$$A_n = A(x_1, x_2, \ldots\ldots\ldots, x_n) = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\& \quad G_n = G(x_1, x_2, \ldots\ldots\ldots, x_n) = (\prod_{i=1}^{n} x_i)^{1/n}$$

The common value of them is the value of μ.

2.3. If the series is found to converge but fail to yield a common converging point for the available data then it is to be understood that the data are insufficient for obtaining the value of μ.

2.4. If the series is found either not to converge or to converge to different points then it is to be understood that the errors involved in the data are not only due to chance but also due to some assignable cause(s). Consequently, the data do not follow the model described by the equation (2.1). Accordingly, the value of μ cannot be determined from the given data in this case.

3. Application to Numerical Data:

Observed data considered here are the data on each of annual maximum & annual minimum of ambient air temperature, occurred in temperature periodic year (TPR), at Guwahati during1969 □ 2013. Application of the method developed here has been shown below in evaluating the central tendency of each of annual maximum & annual minimum of ambient air temperature at Guwahati from these data.

3.1. Annual Maximum Temperature at Guwahati:

The following table shows the observed data on annual maximum of ambient air temperature, occurred in temperature periodic year (TPR), at Guwahati during1969 □ 2013:

Table–3.1.1

Observed Value on Annual Maximum of Ambient Air Temperature (in Degree Celsius)

TPR No (i)	Observed Value (x_i)	Date of occurrence	TPR No (i)	Observed Value (X_i)	Date of occurrence
1	37.1	1969, May, 20	23	37.4	1991, July, 20
2	36.6	1970, April,01	24	39.4	1992, April, 16
3	36.0	1971, March, 27	25	36.4	1993, September, 08
4	35.7	1972, July, 14	26	38.1	1994, May, 07
5	39.0	1973, April,10	27	36.3	1995, May, 14
6	36.1	1974, August, 14	28	39.9	1996, April, 19
7	39.2	1975, April, 10	29	37.4	1998, May 21
8	39.0	1976, April,17	30	37.5	2000, May, 13
9	35.3	1977, August, 14	31	36.7	2001, April,07
10	36.8	1978, May, 19	32	35.7	2002, September, 20
11	38.6	1979, March, 27	33	37.4	2003, July, 26
12	35.1	1980, July, 01	34	38.0	2004, March, 28
13	35.8	1981, June, 21	35	36.6	2005, July, 30
14	36.5	1982, May, 26	36	38.0	2006, August, 11
15	36.7	1983, April, 06	37	37.3	2007, May, 06
16	37.2	1984, April, 06	38	37.3	2008, August, 08
17	36.5	1985, April,26	39	38.0	2009, May, 23
18	38.4	1986, April,03	40	37.2	2010, July, 03
19	37.2	1987, May, 19	41	37.3	2011, August, 30
20	36.4	1988, August, 03	42	37.4	2012, April,03
21	36.7	1989, July, 23	43	38.8	2013, June, 12
22	36.0	1990, September, 02			

Here the observed values x_i (i = 1 , 2 , 3 , ... , 43) can be assumed to be composed of a parameter μ (representing the central tendency of annual maximum) and chance errors.

Determination of Value of μ (the central tendency of annual maximum)

The computed values of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ & $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ have been shown in **Table–3.1.2**.

In **Table–3.1.2**, it is found that the values of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ and $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ are approaching 37.2.

Hence, the true value of the central tendency of annual maximum of the ambient air temperature at Guwahati can be taken as 37.2 Degree Celsius (being the common value of them).

Table–3.1.2

TPR No (n)	Values of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$	Values of $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$
1	37.1	37.1000000000000000000000000001
2	36.85	36.849151957677397816492457471162
3	36.5666666666667	36.563898828796401481877921649392
4	36.35	36.345983740858013867930065117099
5	36.88	36.861929466436019045199560948853
6	36.75	36.733833524906529592733500969237
7	37.1	37.076408237779582297449661223967
8	37.3375	37.311570055521037921393145135076
9	37.1111111111111	37.082517567449318065559488651693
10	37.08	37.054168483041978324085190136495
11	37.21818181818182	37.192102443715679881528536182712
12	37.0416666666667	37.013097252648824947218023353606
13	36.94615384615385	36.918340130784241941575383015056
14	39.75492307692308	36.888300330268507049140494882929
15	36.9	36.875716972191085058307452585677
16	36.91875	36.895901586189937568294003784643
17	36.89411764705882	36.872494839717900572818157696334
18	36.97777777777778	36.955739386544954474314437641907
19	36.98947368421053	36.968555130673424804462730928845
20	36.96	36.939917603077075159918808821998

Table–3.1.2 Continued

TPR No (n)	Values of $A_n = A(x_1, x_2, \ldots\ldots\ldots\ldots, x_n)$	Values of $G_n = G(x_1, x_2, \ldots\ldots\ldots\ldots, x_n)$
21	36.94761904761905	36.9284574714952278715732526211 82
22	36.90454545454545	36.885739991394209779636891882147
23	36.92608695652174	36.907951375530228273997704899203
24	37.02916666666667	37.008568334146486997509657949442
25	37.004	36.984031371155694976981399533444
26	37.04615384615385	37.026342595226743557691601271825
27	37.01851851851852	36.999183617772609404713638815137
28	37.12142857142857	37.099058031246057185229984821213
29	37.13103448275862	37.109394916981577631613488308734
30	37.14333333333333	37.122349300390835138945562995519
31	37.12903225806452	37.108649562396908895644952409924
32	37.084375	37.063799115887290185156645453858
33	37.09393939393939	37.073942480150612130026293039573
34	37.12058823529412	37.10085461802179646288605030756
35	37.10571428571429	37.0864498129419154156748278045
36	37.13055555555556	37.111527173121534380089866586791
37	37.13513513513514	37.116608490052709885185023589966
38	37.13947368421053	37.121423011021134475117582568853
39	37.16153846153846	37.143694849862191868851959925933
40	37.1625	37.145101439431196544614817207745
41	37.16585365853659	37.148871788937429088904002963936
42	37.17142857142857	37.154831390672328678435909684177
43	37.2093023255814	37.192287148576076781925812747586

3.2. Annual Minimum Temperature at Guwahati:

The following table (**Table–3.2.1**) shows the observed data on annual maximum of ambient air temperature, occurred in temperature periodic year (TPR), at Guwahati during 1969 □ 2013.

As earlier, the observed values

$$x_i \; (i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, 43)$$

can in this case also be assumed to be composed of a parameter μ (representing the central tendency of annual minimum) and chance errors.

Table–3.2.1

Observed Value on Annual Minimum of Ambient Air Temperature (in Degree Celsius)

TPR No (i)	Observed Value (x_i)	Date of occurrence	TPR No (i)	Observed Value (X_i)	Date of occurrence
1	6.6	1969, December, 27	23	5.9	1992, January, 05
2	5.9	1971, January, 31	24	8.4	1993, February, 23
3	8.2	1972, January,21	25	7.8	1993, December, 27
4	5.0	1973, February, 03	26	7.5	1995, January, 22
5	6.3	1974, February, 07	27	9.4	1996, January, 19
6	7.4	1975, January, 19	28	Not Available	Not Available
7	6.6	1976, January, 22	29	Not Available	Not Available
8	6.2	1977, January, 30	30	Not Available	Not Available
9	7.3	1978, January, 12	31	8.3	2000, February, 28
10	6.2	1979, January, 09	32	8.9	2001, January, 08
11	6.4	1980, February, 08	33	8.6	2002, January, 26
12	7.5	1981, January, 10	34	8.0	2003, January, 16
13	8.3	1982, February, 07	35	7.9	2004, February, 04
14	4.9	1983, January, 06	36	6.7	2004, December, 27
15	6.1	1984, January, 30	37	9.6	2006, January, 12
16	7.8	1985, January, 19	38	6.4	2007, January, 18
17	8.6	1986, January, 20	39	7.8	2008, February, 03
18	7.7	1987, January, 05	40	9.9	2009, January, 07
19	9.2	1988, January, 01	41	8.6	2010, January , 03
20	6.7	1989, January, 14	42	7.0	2011, January, 21
21	8.6	1989, December, 31	43	6.4	2012, January, 15
22	7.4	1991, January, 20	44	5.6	2013, January, 11

Determination of Value of μ (the central tendency of annual minimum)

The computed values of

$$A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n) \text{ and } G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$$

have been shown in **Table–3.2.2**.

In **Table–3.2.2**, it is found that the values of

$$A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n) \text{ and } G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$$

re not approaching a common value.

Thus, either the data are insufficient to yield the true value of the central tendency of annual minimum of the ambient air temperature at Guwahati or the data do not follow the model described by equation (2.1).

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Table–3.2.2

TPR No (n)	Values of $A_n = A(x_1, x_2, \ldots\ldots\ldots, x_n)$	Values of $G_n = G(x_1, x_2, \ldots\ldots\ldots, x_n)$
1	6.6	6.600000000000000000000000000002
2	6.6	6.240192304729077818454074162669
3	6.366666666666667	6.8349697977658553153553991473341
4	6.825	6.3211333313459467749216627042232
5	6.46	6.3169010013367452786283625052215
6	6.433333333333333	6.4857272185701169855041242093667
7	6.571428571428571	6.5019299609413096599434266552134
8	6.575	6.4633989337912701904703289281854
9	6.533333333333333	6.5514058846887191769812286015146
10	6.61	6.5153871170191872914459649178302
11	6.572727272727273	6.5048119729138514238168851259175
12	6.558333333333333	6.5824407269348502376566800461569
13	6.630769230769231	6.7008890789412278622783766593454
14	6.75	6.5527362805560146596136072863623
15	6.626666666666667	6.5215350514733093207287599447551
16	6.59375	6.5949101184036978619969386660223
17	6.664705882352941	6.6987014182595463410514031559311
18	6.772222222222222	6.7507455267102729305100790423158
19	6.821052631578947	6.8616303970395831831869394230887
20	6.94	6.8534570444635167207666214239384
21	6.928571428571429	6.9279444662370421736229802749321
22	7.004545454545455	6.9487332418478471976809533313006
23	7.021739130434783	6.8994798638029492332170025993497
24	6.975	6.9562840436945652525470112265315
25	7.032	6.9882108302873798619833480810597
26	7.061538461538462	7.0072334421441164904282182533205
27	7.077777777777778	7.0838901531221111364268314501887
28	7.160714285714286	7.1240865180443321452583935939618
32	7.220689655172414	7.1789729488032751962564480429371
33	7.266666666666667	7.2223221258860362445338145542488
34	7.290322580645161	7.2461869803895066094818774956424
35	7.309375	7.2657752520059477722135734563005
36	7.290909090909091	7.247948125913456784586179510479
37	7.358823529411765	7.3081081108055937176215071779575
38	7.331428571428571	7.2804552217016962246905047234237
39	7.344444444444444	7.2944086947827372183189849879502
40	7.413513513513514	7.3548715998389568402950960158236
41	7.444736842105263	7.8154612929430738975137961946202
42	7.433333333333333	7.7934100068228333088588886838578
43	7.4075	7.7551256022354259620074788758758
44	7.363414634146341	7.6937848209202825772540315655963

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Note: Data corresponding to TPR 29, 30 &31 are not available. Therefore, the values of

$$A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n) \text{ and } G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$$

could not be computed corresponding these three TPR.

4. Conclusion:

The method, developed here, can provide the value of the parameter if the data follow the model described by equation (2.1) and if the data size is sufficiently large for obtaining the common converging point of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ and $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$. Conversely, if the common converging point of $A_n = A(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ and $G_n = G(x_1 , x_2 , \ldots\ldots\ldots , x_n)$ is not achieved from the set of data then it implies that either the data do not follow the model described by equation (2.1) or the data size is not sufficient to yield the common converging point.

Regarding the findings obtained on annual maximum and annual minimum of ambient air temperature at Guwahati, the following conclusion can be drawn:

4.1. The central tendency of Annual Maximum of Ambient Air Temperature at Guwahati can be taken as 37.2 Degree Celsius since all the methods applied have yielded the same numerical results and thus the corresponding data can be treated to follow the model described by equation (2.1).

4.2. The central tendency of Annual Minimum of Ambient Air Temperature at Guwahati is not determinable since the methods applied have yielded different numerical results and thus the corresponding data cannot be treated to follow the model described by equation (2.1).

In this connection, it is to be mentioned that the central tendency of each of annual maximum and annual minimum of the ambient air temperature at Guwahati has been evaluated by another method already developed (Chakrabarty , 2017). The findings obtained by that method have been found to be identical with the findings obtained by the method developed here.

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