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Numerical Data Containing One Parameter and Chance Error: Evaluation of the Parameter by Convergence of Statistic

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Abstract

For the purpose of determining error-free estimate i.e. the true value of the parameter from observed data containing the parameter itself and chance error, one method has been developed for determining the value of the same since the existing statistical methods of estimation in such situation fail in finding out of such value of the parameter. The method developed here is based on the convergence of statistic i.e. some function of numerical data. The method has been applied in determining the central tendency of each of annual maximum and annual minimum of the ambient air temperature at Guwahati.

Key words: *1. Parameter; 2. chance error, 3. observed numerical data; 4. convergence of statistic; 5. determination of parameter*

1. Introduction

There are many situations where observations

$$X_1 , X_2 , \dots\dots\dots\dots , X_n$$

are composed of some parameter and chance errors i.e.

$$X_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \dots\dots\dots\dots , n) \qquad (1.1)$$

where (i) μ is the parameter

& (ii) ε_i *is* the chance error associated with X_i

[Chakrabarty (2014*a*, 2014*b*, 2014*c*, 2015*a*, 2015*b*, 2015*c*, 2015*d*, 2015*e*].

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Here,

$$\mu = T(X_i) \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n) \qquad (1.2)$$

where $T(X_i)$ is the true part of X_i.

The existing methods of estimation namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square [Aldrich,1930; Anders ,1999; Barnard,1949; Birnbaum,1962; Ivory,1825; Kendall & Stuart, 1977; Lehmann & Casella George,1998; Lucien, 1990; Walker & Lev,1965] provides

$$\bar{X} = (1/n) \sum_{i=1}^{n} X_i \qquad (1.3)$$

as estimator of the parameter μ.

On the other hand, equation (1.1) implies that

$$\sum_{i=1}^{n} X_i = n\mu + \sum_{i=1}^{n} \varepsilon_i$$

which further implies

$$\bar{X} = \mu + \bar{\varepsilon} \qquad (1.4)$$

where

$$\bar{\varepsilon} = (1/n) \sum_{i=1}^{n} \varepsilon_i \qquad (1.5)$$

Thus, the estimator $\bar{X}$ of the parameter μ suffers from an error $\bar{\varepsilon}$ given by (1.5).

This error however may need not necessarily be zero.

In other words, none of these methods can provide the true value of the parameter μ.

Recently, some studies have been done on determining the true value of the parameter μ involved in the model described by (1.1) [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*a* , 2015*b* , 2015*c* , 2015*d* , 2015*e* , 2015*f* , 2016*a* , 2016*b*) ;Bordoloi & Chakrabarty (2015 , 2015 – 16 , 2016*a* , 2016*b* , 2016*c* , 2016 – 17)]. In the studies some methods have been developed for determining the true value of the parameter μ when

ε_i occurs due to chance only. One of them is based on computing sequence of interval value of μ with decreasing length of interval and then to find out the shortest interval value of μ [Chakrabarty (2014*a* , 2014*b* , 2014*c* , 2015*d*) ; Bordoloi & Chakrabarty (2016*a* , 2016*b*]. The other one is based on stable mid range and median. However, these methods may not be always successful in determining the true value μ [Chakrabarty ,2015*b* ; Bordoloi & Chakrabarty,2015)]. For this reason, attempt has here been made on searching for one more method of determining the true value μ. The method derived here is based on the convergence of statistic i.e. some function of the available numerical data. This paper is based on the derivation of this method and on one numerical application of the method in determining the central tendency of each of annual maximum and annual minimum of ambient air temperature at Guwahati.

2. Method of Determination of Parameter μ

The model considered here is

$$X_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n) \qquad (2.1)$$

where

(i) X_1 , X_2 , ………… , X_n are observed data on the variable X ,

(ii) μ is the parameter

and (iii) ε_i is the chance error associated with X_i .

2.1. Derivation of Method

From Equations described by (2.1),

$$|X_i|\, X_i = \mu\, |X_i| + \varepsilon_i\, |X_i| \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n) \qquad (2.1.1)$$

which implies,

$$\sum_{i=1}^{n} |X_i|\, X_i = \mu \sum_{i=1}^{n} |X_i| + \sum_{i=1}^{n} \varepsilon_i\, |X_i|$$

which implies,

$$\mu = \left(\sum_{i=1}^{n} |X_i|\, X_i\right) / \left(\sum_{i=1}^{n} |X_i|\right) + \left(\sum_{i=1}^{n} \varepsilon_i\, |X_i|\right) / \left(\sum_{i=1}^{n} |X_i|\right) \qquad (2.1.2)$$

Now,

$$\varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , n)$$

being chance errors (i.e. random errors), assume both positive and negative values in random manner.

Hence, the series $\{ U_n \}$

where $U_n = \sum_{i=1}^{n} \varepsilon_i\, |X_i|$

cannot be cannot be monotonically increasing but oscillatory.

Also, the series $\{ V_n \}$

where $V_n = \sum_{i=1}^{n} |X_i|$

is strictly monotonically increasing.

Hence, the series $\{ Z_n \}$ converges to 0 as $n \to \infty$

where $Z_n = U_n / V_n$

$(n = 1 , 2 , 3 , 4 , \ldots\ldots\ldots\ldots)$

i.e. $(U_n / V_n) \to 0$ as $n \to \infty$

i.e. $$\left\{\left(\sum_{i=1}^{n} |X_i|\, \varepsilon_i\right) / \left(\sum_{i=1}^{n} |X_i|\right)\right\} \to 0 \text{ as } n \to \infty \qquad (2.1.3)$$

Accordingly,

$$\left\{\left(\sum_{i=1}^{n} |X_i|\, X_i\right) / \left(\sum_{i=1}^{n} |X_i|\right)\right\} \to \mu \text{ as } n \to \infty \qquad (2.1.4)$$

Thus, the value towards which the computed values of

$$\{(\sum_{i=1}^{n} |X_i|\, X_i\,)/(\sum_{i=1}^{n} |X_i|)\} \qquad (2.1.5)$$

converge is the value of μ.

Again from (2.1),

$$X_i^{\,3} = \mu\, X_i^{\,2} + \varepsilon_i X_i^{\,2}\ , \quad (i = 1\ , 2\ , \ldots\ldots\ldots\ldots\ , n) \qquad (2.1.6)$$

which implies,

$$\sum_{i=1}^{n} X_i^{\,3} = \mu \sum_{i=1}^{n} X_i^{\,2} + \sum_{i=1}^{n} \varepsilon_i X_i^{\,2}$$

which further implies,

$$\mu = (\sum_{i=1}^{n} X_i^{\,3})/(\sum_{i=1}^{n} X_i^{\,2}) + (\sum_{i=1}^{n} \varepsilon_i X_i^{\,2})/(\sum_{i=1}^{n} X_i^{\,2}) \qquad (2.1.7)$$

As earlier the series

$$\{\ \sum_{i=1}^{n} \varepsilon_i X_i^{\,2}\ \}$$

cannot be monotonically increasing but oscillatory.

Also, the series

$$\{\ \sum_{i=1}^{n} X_i^{\,2}\ \}$$

is strictly monotonically increasing.

Hence,

$$\{(\sum_{i=1}^{n} \varepsilon_i X_i^{\,2})/(\sum_{i=1}^{n} X_i^{\,2})\} \to 0 \ \text{ as } \ n \to \infty \qquad (2.1.9)$$

Accordingly,

$$\{(\sum_{i=1}^{n} X_i^{\,3})/(\sum_{i=1}^{n} X_i^{\,2})\} \to \mu \ \text{ as } \ n \to \infty \qquad (2.1.10)$$

Thus, the value towards which the computed values of

$$\{(\sum_{i=1}^{n} X_i^{\,3})/(\sum_{i=1}^{n} X_i^{\,2})\} \qquad (2.1.11)$$

converge is the value of μ.

Similarly, it can be shown that

$$\{(\sum_{i=1}^{n} X_i^{\,5})/(\sum_{i=1}^{n} X_i^{\,4})\} \rightarrow \mu \quad \text{as} \quad n \rightarrow \infty \qquad (2.1.12)$$

which means that the value towards which the computed values of

$$\{(\sum_{i=1}^{n} X_i^{\,5})/(\sum_{i=1}^{n} X_i^{\,4})\}$$

converge is the value of μ

i.e. $$\{(\sum_{i=1}^{n} X_i^{\,5})/(\sum_{i=1}^{n} X_i^{\,4})\} \rightarrow \mu \quad \text{as} \quad n \rightarrow \infty \qquad (2.1.13)$$

In general,

$$\{(\sum_{i=1}^{n} X_i^{\,2k+1})/(\sum_{i=1}^{n} X_i^{\,2k})\} \rightarrow \mu \quad \text{as} \quad n \rightarrow \infty \qquad (2.1.14)$$

(for $k = 1, 2, 3,$ ....).

which means that the value towards which the computed values of

$$\{(\sum_{i=1}^{n} X_i^{\,2k+1})/(\sum_{i=1}^{n} X_i^{\,2k})\} \qquad (2.1.15)$$

converge is the value of μ

(for $k = 1, 2, 3,$ ....).

Note:

(1) In order to determine the value of μ, it is required to compute the converging values of at least two of the series

$$\{(\sum_{i=1}^{n} |X_i|\, X_i)/(\sum_{i=1}^{n} |X_i|)\}\ ,$$

$$\{(\sum_{i=1}^{n} X_i^{\,3})/(\sum_{i=1}^{n} X_i^{\,2})\}\ ,$$

$$\{(\sum_{i=1}^{n} X_i^{\,5})/(\sum_{i=1}^{n} X_i^{\,4})\}$$

etc.

The common value of them is the value of μ.

(2) If the series are found to converge but fail to yield a common converging point for the available data then it is to be understood that the data are insufficient for obtaining the value of μ.

(3) If the series are found either not to converge or to converge to different points then it is to be understood that the errors involved in the data are not only due to chance but also due to some assignable cause(s). Consequently, the data do not follow the model described by (2.1). In this case the value of μ cannot be determined from the given data.

3. Application to Numerical Data

Numerical data considered here are the data on each of annual maximum & annual minimum of ambient air temperature at Guwahati.

3.1. Annual Maximum Temperature at Guwahati

The following table shows the data on annual maximum of ambient air temperature, occurred in temperature periodic year (TPR), at Guwahati:

Table-3.1.1

Observed Value on Annual Maximum of Ambient Air Temperature (in Degree Celsius)

TPR No (i)	Observed Value (X_i)	Date of occurrence	TPR No (i)	Observed Value (X_i)	Date of occurrence
1	37.1	1969, May, 20	23	37.4	1991, July, 20
2	36.6	1970, April,01	24	39.4	1992, April, 16
3	36.0	1971, March, 27	25	36.4	1993, September, 08
4	35.7	1972, July, 14	26	38.1	1994, May, 07
5	39.0	1973, April,10	27	36.3	1995, May, 14
6	36.1	1974, August, 14	28	39.9	1996, April, 19
7	39.2	1975, April, 10	29	37.4	1998, May 21
8	39.0	1976, April,17	30	37.5	2000, May, 13
9	35.3	1977, August, 14	31	36.7	2001, April,07
10	36.8	1978, May, 19	32	35.7	2002, September, 20
11	38.6	1979, March, 27	33	37.4	2003, July, 26
12	35.1	1980, July, 01	34	38.0	2004, March, 28
13	35.8	1981, June, 21	35	36.6	2005, July, 30
14	36.5	1982, May, 26	36	38.0	2006, August, 11
15	36.7	1983, April, 06	37	37.3	2007, May, 06
16	37.2	1984, April, 06	38	37.3	2008, August, 08
17	36.5	1985, April,26	39	38.0	2009, May, 23
18	38.4	1986, April,03	40	37.2	2010, July, 03
19	37.2	1987, May, 19	41	37.3	2011, August, 30
20	36.4	1988, August, 03	42	37.4	2012, April,03
21	36.7	1989, July, 23	43	38.8	2013, June, 12
22	36.0	1990, September, 02			

Here, data are assumed to follow the model

$$X_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , 43)$$

where μ is central tendency of annual maximum and ε_i is the chance error.

Determination of Value of μ (the central tendency of annual maximum)

In order to determine the value of μ, let us construct the following table (Table – 3.1.2):

Table-3.1.2

TPR No (n)	$(1/n)\sum_{i=1}^{N} X_i$	$(\sum_{i=1}^{n} \lvert X_i \rvert X_i)/(\sum_{i=1}^{n} \lvert X_i \rvert)$	$(\sum_{i=1}^{n} X_i^3)/(\sum_{i=1}^{n} X_i^2)$
1	37.1	37.1	37.1
2	36.85	36.8516960651289	36.85339197413815
3	36.56666666666667	36.5721969006381	36.57771792909718
4	36.35	36.35804676753783	36.36611267275265
5	36.88	36.91681127982646	36.95460568258939
6	36.75	36.78308390022676	36.81728895393352
7	37.1	37.14790142472083	37.19681123546356
8	37.3375	37.38972212922665	37.4423855534096
9	37.11111111111111	37.1688622754491	37.22734453537827
10	37.08	37.13225458468177	37.18531227485068
11	37.21818181818182	37.27063996091842	37.32345261033122
12	37.04166666666667	37.09923509561305	37.15733893409643
13	36.94615384615385	37.00239433687279	37.05945488050191
14	39.75492307692308	36.96691176470588	37.02044136198278
15	36.9	36.94921409214092	36.99933769782885
16	36.91875	36.96500761808024	37.01205496804934
17	36.89411764705882	36.93794642857143	36.98260915731879
18	36.97777777777778	37.02229567307692	37.06742469373477
19	36.98947368421053	37.03170176437109	37.07447394670228
20	36.96	37.00059523809524	37.0418003249626
21	36.94761904761905	36.98637711045238	37.0257583787986
22	36.90454545454545	36.94264071930041	36.98143635353122
23	36.92608695652174	36.96278111385847	37.00008635823409
24	37.02916666666667	37.07083380218296	37.11317016164894
25	37.004	37.04443843908767	37.08559714804372
26	37.04615384615385	37.08619186046512	37.12681939855694
27	37.01851851851852	37.0576388194097	37.09740477025501
28	37.12142857142857	37.16675004810468	37.21290128269644
29	37.13103448275862	37.1748514115899	37.21943905402664
30	37.14333333333333	37.18579377187472	37.22896066790021
31	37.12903225806452	37.17030408340573	37.21230799041302
32	37.084375	37.12607230133985	37.16856013878224
33	37.09393939393939	37.13444163058574	37.17587989511678
34	37.12058823529412	37.16050233737422	37.20106161435744
35	37.10571428571429	37.14470624470624	37.18437088190402
36	37.13055555555556	37.16902072267525	37.20807614871881
37	37.13513513513514	37.17257641921397	37.21058016114304
38	37.13947368421053	37.1759441649543	37.21295137212552
39	37.16153846153846	37.19755054164079	37.23403261077035
40	37.1625	37.19761183989236	37.2331808822798
41	37.16585365853659	37.20011812573829	37.23482089083036
42	37.17142857142857	37.20490648219318	37.23879865987883
43	37.2093023255814	37.2435875	37.27823992855146

In Table–3.1.2, it is found that the values of the two functions

$$\left(\sum_{i=1}^{n} |X_i|\, X_i\right)/\left(\sum_{i=1}^{n} |X_i|\right) \quad \& \quad \left(\sum_{i=1}^{n} X_i^{3}\right)/\left(\sum_{i=1}^{n} X_i^{2}\right)$$

are approaching 37.2435875 & 37.27823992855146 respectively. Hence, the true value of the central tendency of annual maximum of the ambient air temperature at Guwahati can be taken as 37.2 Degree Celsius (being the common value of them).

3.2. Annual Minimum Temperature at Guwahati

The following table shows the data on annual maximum of ambient air temperature, occurred in temperature periodic year (TPR), at Guwahati.

Table–3.2.1

Observed Value on Annual Minimum of Ambient Air Temperature (in Degree Celsius)

TPR No (i)	Observed Value (Y_i)	Date of occurrence	TPR No (i)	Observed Value (Y_i)	Date of occurrence
1	6.6	1969, December, 27	23	5.9	1992, January, 05
2	5.9	1971, January, 31 & February, 01	24	8.4	1993, February, 23
3	8.2	1972, January,21	25	7.8	1993, December, 27
4	5.0	1973, February, 03	26	7.5	1995, January, 22
5	6.3	1974, February, 07	27	9.4	1996, January, 19
6	7.4	1975, January, 19	28	NA	NA
7	6.6	1976, January, 22	29	NA	NA
8	6.2	1977, January, 30	30	NA	NA
9	7.3	1978, January, 12	31	8.3	2000, February, 28
10	6.2	1979, January, 09	32	8.9	2001, January, 08
11	6.4	1980, February, 08	33	8.6	2002, January, 26
12	7.5	1981, January, 10	34	8.0	2003, January, 16
13	8.3	1982, February, 07	35	7.9	2004, February, 04
14	4.9	1983, January, 06	36	6.7	2004, December, 27
15	6.1	1984, January, 30	37	9.6	2006, January, 12
16	7.8	1985, January, 19	38	6.4	2007, January, 18
17	8.6	1986, January, 20	39	7.8	2008, February, 03
18	7.7	1987, January, 05	40	9.9	2009, January, 07
19	9.2	1988, January, 01	41	8.6	2010, January , 03
20	6.7	1989, January, 14	42	7.0	2011, January, 21
21	8.6	1989, December, 31	43	6.4	2012, January, 15
22	7.4	1991, January, 20	44	5.6	2013, January, 11

These data are assumed to follow the model

$$Y_i = \mu^{/} + \varepsilon_i^{/} \quad , \quad (i = 1, 2, \ldots\ldots\ldots\ldots, 44)$$

where $\mu^{/}$ is the central tendency of annual minimum and $\varepsilon_i^{/}$ is the chance error.

Determination of Value of $\mu^{/}$ *(the central tendency of annual minimum)*

In order to determine the value of $\mu^{/}$, let us construct the following table (Table – 3.2.2):

Table–3.2.2

TPR No (n)	$(1/n)\sum_{i=1}^{N} Y_i$	$(\sum_{i=1}^{n} \lvert Y_i \rvert Y_i)/(\sum_{i=1}^{n} \lvert Y_i \rvert)$	$(\sum_{i=1}^{n} Y_i^3)/(\sum_{i=1}^{n} Y_i^2)$
1	6.6	6.6	6.6
2	6.6	6.6	6.6
3	6.366666666666667	6.383769633507853	6.400155827113918
4	6.825	6.929304029304029	7.039905904741767
5	6.46	6.630650154798762	6.801788299014801
6	6.433333333333333	6.576683937823834	6.723335696840778
7	6.571428571428571	6.709130434782609	6.843399650055084
8	6.575	6.695437262357414	6.813294338122551
9	6.533333333333333	6.643197278911565	6.752941477650914
10	6.61	6.715733736762481	6.818614133495528
11	6.572727272727273	6.671507607192254	6.769314812895201
12	6.558333333333333	6.649428208386277	6.740408171064952
13	6.630769230769231	6.723433874709977	6.814131410035199
14	6.75	6.861904761904762	6.971987046032848
15	6.626666666666667	6.765191146881288	6.898007316420308
16	6.59375	6.726729857819905	6.856165541730664
17	6.664705882352941	6.800617828773169	6.930691360267875
18	6.772222222222222	6.927563576702215	7.076892015110069
19	6.821052631578947	6.973456790123457	7.117770204479065
20	6.94	7.121037463976945	7.296078510724403
21	6.928571428571429	7.101649484536082	7.270182620561507
22	7.004545454545455	7.185269305645685	7.359009257168661
23	7.021739130434783	7.195108359133127	7.360940955757696
24	6.975	7.149462365591398	7.318448889557327
25	7.032	7.209215017064846	7.378663068692894
26	7.061538461538462	7.234313725490196	7.397962686904278
27	7.077777777777778	7.244740973312402	7.402108388047412
28	7.160714285714286	7.345785536159601	7.521968591079758
29	NA	NA	NA
30	NA	NA	NA
31	NA	NA	NA
32	7.220689655172414	7.411843361986628	7.592297878920646

Table–3.2.2 Continued

TPR No (n)	$(1/n)\sum_{i=1}^{n} Y_i$	$(\sum_{i=1}^{n} \lvert Y_i\rvert Y_i)/(\sum_{i=1}^{n} \lvert Y_i\rvert)$	$(\sum_{i=1}^{n} Y_i^3)/(\sum_{i=1}^{n} Y_i^2)$
33	7.266666666666667	7.458715596330275	7.638134071340713
34	7.290322580645161	7.47787610619469	7.651837869822485
35	7.309375	7.492133390337751	7.660675869231515
36	7.290909090909091	7.470074812967581	7.636681689200467
37	7.358823529411765	7.551798561151079	7.732444190403607
38	7.331428571428571	7.52307092751364	7.704172149066006
39	7.344444444444444	7.531240544629349	7.707100027118508
40	7.413513513513514	7.616733503463361	7.809971425426106
41	7.444736842105263	7.646624248851184	7.836982197917004
42	7.433333333333333	7.631010693342532	7.818443380661143
43	7.4075	7.604421194735066	7.792657964929722
44	7.363414634146341	7.567240808214641	7.762559366177146

NA means Not Available

In Table–3.2.2, it is found that the values of the two functions

$$(\sum_{i=1}^{n} |Y_i|\, X_i)/(\sum_{i=1}^{n} |Y_i|) \quad \& \quad (\sum_{i=1}^{n} Y_i^{3})/(\sum_{i=1}^{n} Y_i^{2})$$

are not to converge to a limit.

Thus, either the data are insufficient to yield the true value of the central tendency of annual maximum of the ambient air temperature at Guwahati or the data do not follow the model described by equation (2.1).

4. Conclusion

In this study, one method has been searched for determining the true value of the parameter from observed data containing the parameter itself and chance error i.e. from the data following the model described by equation (2.1) since the existing statistical methods of estimation fail in finding out of such value of the parameter in such situation. The method derived here is based on the convergence of statistic i.e. some function of numerical data.

The method can successfully yield the value of the parameter if the data follow the model described by equation (2.1) and if the data size is sufficient i.e. if the convergence of statistic is achieved. Conversely, if the convergence of statistic is not achieved from the set of data then it implies that either the data do not follow the model described by equation (2.1) or the data size is not sufficient to yield convergence of statistic.

Regarding the findings obtained on annual maximum and annual minimum of ambient air temperature at Guwahati, the following conclusion can be drawn:

(1) The central tendency of Annual Maximum of Ambient Air Temperature at Guwahati can be taken as 37.2 Degree Celsius since all the methods applied have yielded the same numerical results and thus the corresponding data can be treated to follow the model described by equation (2.1).

(2) The central tendency of Annual Minimum of Ambient Air Temperature at Guwahati is not determinable since the methods applied have yielded different numerical results and thus the corresponding data cannot be treated to follow the model described by equation (2.1).

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