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Analysis of Errors Associated to Observations of Measurement Type

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Abstract

Observations or data, collected from an experiment which is free from all sorts of assignable errors suffer from chance error (which is unavoidable or uncontrollable). Consequently the findings obtained by analyzing the observations which are free from the assignable errors are also subject to errors due to the presence of chance error in the observations. Determination of constant(s) associated to mathematical model(s), in different situations, based on the observations is also subject to error due to the same reason. This paper is based on the mathematical model(s) that have been identified for describing the association of chance error(s) in determining constant(s) in some distinct situations where observations/data are of measurement type. Also, chance error has been analyzed in the simplest situation as identified.

Key Words: Observation of measurement type; chance error; mathematical model; constant determination.

1. Introduction

Observations or data, collected from experiment or survey, normally suffer from various types of errors. Error occurs due to many causes. These causes can be broadly divided into two types namely

1. Assignable cause that is avoidable / controllable

& 2. Chance cause that is unavoidable / uncontrollable.

Even if all the assignable causes of error are controlled or eliminated, observations still do not become free from error. Each of them still suffers from some error which occurs due to some unknown and unintentional cause that is nothing but the chance cause. Findings obtained by analyzing the observations or data which are free from the assignable errors are also subject to errors due to the presence of chance error in the observations. Determination of constant(s), in different

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situations, based on such observations is also subject to error due to the same reason. This leads to the necessity of searching for the association/connection of chance error with observation/data. In the current study, attempt has been made on this aspect of chance error. Mathematical models have been identified for describing the association of chance error in determining constant(s) in some distinct situations where observations/data are of measurement type. Method of determination of constant has been searched for by analyzing chance error in the situation where observations/data consist of itself and chance error.

2. Gaussian Discovery

In the year 1809, German mathematician Carl Friedrich Gauss discovered the most significant probability distribution in the theory of statistics popularly known as normal distribution, the credit for which discovery is also given by some authors to a French mathematician Abraham De Moivre who published a paper in 1738 that showed the normal distribution as an approximation to the binomial distribution discovered by James Bernoulli in 1713 [De Moivre, 1711 ; Bernoulli, 1713 ; De Moivre, 1718 ; Walker and Lev, 1965 ; Kendall and Stuart, 1977 & 1979 ; Stigler, 1982 ; Walker, 1985 ; Brye, 1995 ; Hazewinkel, 2001 ; Marsagilia, 2004 ; Chakrabarty, 2005 & 2008]. The normal probability distribution plays the key role in the theory of statistics as well as in the application of statistics. There are innumerable situations where one can think of applying the theory of normal probability distribution to handle the situations.

The probability density function of normal probability distribution discovered by *Gauss* is described by the probability density function

$$f(x : \mu , \sigma) = \{\sigma (2\Pi)^{½}\}^{-1} . exp [- ½ \{(x - \mu)/\sigma\}^2] , \quad (1)$$

$$-\infty < x < \infty , \quad -\infty < \mu < \infty , \quad 0 < \sigma < \infty .$$

where (i) X is the associated normal variable,

(ii) μ & σ are the two parameters of the distribution

and (iii) Mean of $X = \mu$ & Standard Deviation of $X = \sigma$.

Note: If $\mu = 0$ & $\sigma = 1$,

the density is standardized and X then becomes a standard normal variable.

2.1. Area Property of Gaussian Distribution

If $X \sim N(\mu , \sigma)$, then

$$\text{(i) } P(\mu - 1.96\, \sigma < X < \mu - 1.96\, \sigma) = 0.95, \quad (2)$$

$$\text{(ii) } P(\mu - 2.58\,\sigma < X < \mu - 2.58\,\sigma) = 0.99 \quad (3)$$

$$\&\ \text{(iii) } P(\mu - 3\,\sigma < X < \mu - 3\,\sigma) = 0.9973\,. \quad (4)$$

If X is a standard normal variable then

$$\text{(i) } P(-1.96 < X < 1.96) = 0.95, \quad (5)$$

$$\text{(ii) } P(-2.58 < X < 2.58) = 0.99 \quad (6)$$

$$\&\ \text{(iii) } P(-3 < X < 3) = 0.9973\,. \quad (7)$$

3. Errors in Experimental Observations

Situations associated to different experiments are different. There are innumerable experiments which can be placed in one of the following situations:

Situation – 1

In some situations, observation consists of true value of a constant μ whose value is unknown and to be determined on the basis of observed data.

Let X_1 , X_2 , ………… , X_n be n random observations on μ obtained from an experiment which is free from all sorts of assignable errors. Here, the observations are on μ. Therefore, they should be equal and this common value is nothing but the true value of μ. Since they are different, there exists some cause behind the observations being different. This cause is nothing but the chance cause of error due to which the observations have been compelled to be different. Thus in this situation, each observation X_i is composed of true value of μ and an error ε_i .

Thus the observations, in such types of practical situations, satisfy the model

$$X_i = T(\mu) + \varepsilon_i\ , \qquad (i = 1\,, 2\,, \ldots\ldots\ldots\,, n)$$

where (i) X_i is the i^{th} observation on μ,

(ii) $T(\mu)$ is the true value of μ

& (iii) ε_i is the error associated to X_i .

It is to be noted that the true part of an observation is nothing but the true value of the constant μ and the observations are collected for determining its true value.

Situation – 2

In some situation, random observations on a pair (X , Y) of two variables X and Y namely

$$(X_1\,, Y_1)\,, (X_2\,, Y_2)\,, \ldots\ldots\ldots\ldots\ldots\ldots\,, (X_n\,, Y_n)$$

are collected to determine the value of an constant μ which connects the two variables X and Y by the theoretical relationship

$$f(X) = \mu\, f(Y) \quad \text{i.e.} \quad \mu = f(X) \,/\, \mu\, f(Y)$$

for some function $f(.)$.

In this situation, each observation on each of the two variables is influenced by the chance error due to the similar cause mentioned in Situation – 1.

Thus the observations, in such types of practical situations, satisfy the model

$$X_i = T_i + \varepsilon_i$$

$$\& \; Y_i = T_i^{/} + \varepsilon_i^{/}$$

$$(i = 1 , 2 , \ldots\ldots\ldots , n)$$

where (i) T_i is the true part of X_i,

(ii) ε_i is the error associated to X_i,

(iii) $T_i^{/}$ is the true part of Y_i

& (iv) $\varepsilon_i^{/}$ is the error associated to Y_i .

Consequently, for the constant μ the model becomes

$$f(T_i) = \mu \, f(T_i^{/}) \quad \text{i.e.} \quad \mu = f(T_i) \,/\, f(T_i^{/})$$

$$\text{which means} \quad f(X_i - \varepsilon_i) = \mu \, f(Y_i^{/} - \varepsilon_i^{/}) \quad \text{i.e.} \quad \mu = f(X_i - \varepsilon_i) \,/\, f(Y_i^{/} - \varepsilon_i^{/})$$

$$(i = 1 , 2 , \ldots\ldots\ldots , n).$$

Thus, the true value of $\mu = f(T_i) \,/\, f(T_i^{/}) = f(X_i - \varepsilon_i) \,/\, f(Y_i^{/} - \varepsilon_i^{/})$.

But, the values of μ one obtains from the observations are $f(X_i) \,/\, f(Y_i)$ ($i = 1 , 2 , \ldots\ldots\ldots , n$), which are subject to errors.

Situation – 3

In Situation – 2, it may be so that the constant μ connects the two variables X and Y by the theoretical relationship

$$f(X) = \mu \, g(Y)$$

for some functions $f(.)$ and $g(.)$.

In such types of practical situations, the model will be

$$f(T_i) = \mu \, g(T_i^{/}) \quad \text{i.e.} \quad \mu = f(T_i) \,/\, g(T_i^{/})$$

$$\text{which means} \quad f(X_i - \varepsilon_i) = \mu \, g(Y_i^{/} - \varepsilon_i^{/}) \quad \text{i.e.} \quad \mu = f(X_i - \varepsilon_i) \,/\, g(Y_i^{/} - \varepsilon_i^{/})$$

$$(i = 1 , 2 , \ldots\ldots\ldots , n).$$

Thus in this situation,

the true value of $\mu = f(T_i) \,/\, g(T_i^{/}) = f(X_i - \varepsilon_i) \,/\, g(Y_i^{/} - \varepsilon_i^{/})$.

But, the values of μ one obtains from the observations are $f(X_i) \,/\, g(Y_i)$ ($i = 1 , 2 , \ldots\ldots\ldots , n$), which are subject to errors.

Situation – 4

In Situation – 2, it may be so that the variable Y theoretically depends linearly upon the variable X i.e.

$$Y = \alpha + \beta X$$

where α & β are two constants.

In such types of practical situations,

$$T_i^{/} = \alpha + \beta T_i \quad \text{i.e.} \quad Y_i^{/} - \varepsilon_i^{\,/} = \alpha + \beta.(X_i - \varepsilon_i) \quad , \quad (i = 1, 2, \ldots\ldots\ldots, n).$$

Situation – 5

In Situation – 2, it may be so that the variable Y theoretically depends quadratically upon the variable X i.e.

$$Y = \alpha + \beta X + \gamma X^2$$

where α, β & γ are three constants.

In such types of practical situations,

$$T_i^{/} = \alpha + \beta T_i + \gamma T_i^{\,2} \quad \text{i.e.} \quad Y_i - \varepsilon_i^{\,/} = \alpha + \beta.(X_i - \varepsilon_i) + \gamma.(X_i - \varepsilon_i)^2 \quad , \quad (i = 1, 2, \ldots\ldots\ldots, n).$$

Situation – 6 (Generalized Situation of Situation - 4 & Situation – 5)

The variable Y theoretically depends upon the variable X by the polynomial of degree p of the form

$$Y = a_0 + a_1 X + a_2 X^2 + \ldots\ldots\ldots\ldots + a_p X^p$$

where a_0 , a_1 , a_2 , ………… , a_p are constants.

In such types of practical situations, the model will be

$$T_i^{/} = a_0 + a_1 T_i + a_2 T_i^{\,2} + \ldots\ldots\ldots\ldots + a_p T_i^{\,p}$$

$$\text{i.e.} \quad Y_i - \varepsilon_i^{\,/} = a_0 + a_1.(X_i - \varepsilon_i) + a_2.(X_i - \varepsilon_i)^2 + \ldots\ldots\ldots\ldots + a_p.(X_i - \varepsilon_i)^p$$

$$(i = 1, 2, \ldots\ldots\ldots, n).$$

Note (Some More Situations)

Mathematical models have been identified for describing the association of chance error in determining constants in some more distinct situations. Those, along with the above, have been summarized below (Table – I).

Table – I

Association of Chance Error with the Constant(s) in Different Situations

Serial No	Theoretical relationship	Constant(s)	Relationship among constants, observations and errors
1.	$f(X) = \mu\, f(Y)$	μ	$f(X_i - \varepsilon_i) = \mu\, f(Y_i^{/} - \varepsilon_i^{/})$
2.	$f(X) = \mu\, g(Y)$	μ	$f(X_i - \varepsilon_i) = \mu\, g(Y_i^{/} - \varepsilon_i^{/})$
3.	$Y = \alpha + \beta X$	α , β	$Y_i - \varepsilon_i^{/} = \alpha + \beta.(X_i - \varepsilon_i)$
4.	$Y = \alpha + \beta X + \gamma X^2$	α , β , γ	$Y_i - \varepsilon_i^{/} = \alpha + \beta.(X_i - \varepsilon_i) + \gamma.(X_i - \varepsilon_i)^2$
5.	$Y = a_0 + a_1 X + a_2 X^2 + \ldots\ldots\ldots\ldots + a_p X^p$	$a_0 , a_1 , a_2 , \ldots\ldots , a_p$	$Y_i - \varepsilon_i^{/} = a_0 + a_1.(X_i - \varepsilon_i) + a_2.(X_i - \varepsilon_i)^2 + \ldots\ldots + a_p.(X_i - \varepsilon_i)^p$
6.	$f(X , Y) = \mu$	μ	$f(Y_i - \varepsilon_i^{/} ,\ X_i - \varepsilon_i) = \mu$
7.	$Y = \lambda\, exp\,(-\nu X)$	λ , ν	$Y_i - \varepsilon_i^{/} = \lambda\, exp\{-\nu\ (X_i - \varepsilon_i)\}$
8.	$Y = \lambda\, exp\,(-\nu X^{-1})$	λ , ν	$Y_i - \varepsilon_i^{/} = \lambda\, exp\{-\nu\ (X_i - \varepsilon_i)^{-1}\}$
9.	$Y = \mu + \lambda\, exp\,(-\nu X)$	μ , λ , ν	$Y_i - \varepsilon_i^{/} = \mu + \lambda\, exp\{-\nu\ (X_i - \varepsilon_i)\}$
10.	$Y = \mu + \lambda\, exp\,(-\nu X^{-1})$	μ , λ , ν	$Y_i - \varepsilon_i^{/} = \mu + \lambda\, exp\{-\nu\ (X_i - \varepsilon_i)^{-1}\}$
11.	$Y = \{\alpha + \beta\, exp\,(\gamma X)\}^{-1}$	α , β , γ	$Y_i - \varepsilon_i^{/} = [\alpha + \beta\, exp\{\gamma(X_i - \varepsilon_i)]^{-1}$
12.	$Y = \mu + \lambda X^{-1}$	μ , λ	$Y_i - \varepsilon_i^{/} = \mu + \lambda\,(X_i - \varepsilon_i)^{-1}$
13.	$Y = f(X : \mu_1 , \mu_2 , \ldots\ldots , \mu_k)$	$\mu_1 , \mu_2 , \ldots\ldots , \mu_k$	$Y_i - \varepsilon_i^{/} = f(X_i - \varepsilon_i : \mu_1 , \mu_2 , \ldots\ldots , \mu_k)$
14.	$h(X , Y) = \mu$	μ	$h(X_i - \varepsilon_i ,\ Y_i - \varepsilon_i^{/}) = \mu$
15.	$Y = \theta\, exp\,\{g(X) : \mu_1 , \mu_2 , \ldots\ldots\ldots , \mu_k)\}$	$\theta , \mu_1 , \mu_2 , \ldots\ldots , \mu_k$	$Y_i - \varepsilon_i^{/} = \theta\, exp\{g(X_i - \varepsilon_i : \mu_1 , \mu_2 , \ldots\ldots\ldots , \mu_k)\}$

In the table, X and Y are two variables where Y theoretically depends upon X and

$$(X_1 , Y_1) , (X_2 , Y_2) , \ldots\ldots\ldots\ldots\ldots\ldots\ldots , (X_n , Y_n)$$

are random observations on the pair (X , Y).

4. Analysis of errors (Situation – 1)

Let us consider Situation – 1 where observations consist of the true value of a constant μ whose value is unknown and is to be determined.

In this situation If $X_1 , X_2 , \ldots\ldots , X_n$ are n random observations on μ, we have

$$X_i = T(\mu) + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots , n) \qquad (8)$$

where (i) X_i is the i^{th} observation on μ,

(ii) $T(\mu)$ is the true value of μ

& (iii) ε_i is the chance error associated to X_i .

Here ε_1 , ε_2 , ……. , ε_n are values of the chance error variable ε associated to X_1 , X_2 , …… , X_n respectively.

It is to be noted that

(1) $X_1 , X_2 , \ldots\ldots , X_n$ are known,

(2) $T(\mu) , \varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_n$ are unknown

& (3) the number of linear equations in (8) is n with $(n + 1)$ unknowns implying that the equations are not solvable mathematically.

Reasonable facts /Assumptions regarding ε_i **:**

(1) $\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots\ldots , \varepsilon_n$ are unknown values of the variables ε.

(2) The values $\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots\ldots , \varepsilon_n$ are very small relative to the respective values $X_1 , X_2 , \ldots\ldots\ldots\ldots , X_n$.

(3) The variable ε can assume both positive and negative values.

(4) $P(-a - da < \varepsilon < -a) = P(a < \varepsilon < a + da)$ for every real a.

(5) $P(a < \varepsilon < a + da) > P(b < \varepsilon < b + db)$

& $P(-a - da < \varepsilon < -a) < P(-b - db < \varepsilon < -b)$

for every real positive $a < b$.

(6) The facts (3), (4) & (5) together imply that ε obeys the normal probability law.

(7) Sum of all possible values of each ε is 0 (zero) which together with the fact (6) implies that $E(\varepsilon) = 0$.

(8) Standard deviation of ε is unknown and small, say σ_ε.

(9) The facts (6), (7) & (8) together imply that ε obeys the normal probability law with mean (expectation) 0 & standard deviation σ_ε. Thus

$$\varepsilon \sim N(0 , \sigma_\varepsilon) \tag{9}$$

4.1. Confidence Interval of Error ‘ε’

Since $\varepsilon \sim N(0 , \sigma_\varepsilon)$, the area property of Gaussian distribution given by the equation.5,is

$$P(-1.96\, \sigma_\varepsilon < \varepsilon < -1.96\, \sigma_\varepsilon) = 0.95 \tag{10}$$

i.e. the interval

$$(-1.96\, \sigma_\varepsilon \;,\; 1.96\, \sigma_\varepsilon) \tag{11}$$

is the 95% confidence interval of ε.

This means that out of 100 random observations on ε (unknown), maximum 5 observations fall outside this interval.

Again by the area property of Gaussian distribution given by the equations (6) and (7),

$$P(-2.58\, \sigma_\varepsilon < \varepsilon < -2.58\, \sigma_\varepsilon) = 0.99 \tag{12}$$

$$\& \ P(-3\sigma_\varepsilon < \varepsilon < 3\sigma_\varepsilon) = 0.9973 \quad (13)$$

respectively which implies that the intervals

$$(-2.58\,\sigma_\varepsilon \quad , \quad 2.58\,\sigma_\varepsilon) \quad (14)$$

$$\& \ (-3\sigma_\varepsilon \quad , \quad 3\sigma_\varepsilon) \quad (15)$$

are respectively the 99% & 99.73% confidence intervals of ε.

These respectively mean that out of 100 random observations on ε (unknown), maximum 1 observation falls outside the interval $(-2.58\,\sigma_\varepsilon , 2.58\,\sigma_\varepsilon)$ and out of 10000 random observations on ε (unknown), maximum 27 observations fall outside the interval $(-3\sigma_\varepsilon , 3\sigma_\varepsilon)$.

4.2. Confidence Interval of the Parameter 'μ'

Also under the assumption numbered (9),

$$X - \mu \sim N(0 , \sigma_\varepsilon).$$

This implies that $X \sim N(\mu , \sigma_\varepsilon)$.

Thus by the same area property of Gaussian distribution mentioned above,

$$P(X - 1.96\,\sigma_\varepsilon < \mu < X - 1.96\,\sigma_\varepsilon) = 0.95, \quad (16)$$

$$P(X - 2.58\,\sigma_\varepsilon < \mu < X - 2.58\,\sigma_\varepsilon) = 0.99 \quad (17)$$

$$\& \ P(X - 3\sigma_\varepsilon < \mu < X - 3\sigma_\varepsilon) = 0.9973 \quad (18)$$

Theses imply the intervals

$$(X - 1.96\,\sigma_\varepsilon \quad , \quad X - 1.96\,\sigma_\varepsilon) , \quad (19)$$

$$(X - 2.58\,\sigma_\varepsilon \quad , \quad X - 2.58\,\sigma_\varepsilon) \quad (20)$$

$$\& \ (X - 3\sigma_\varepsilon \quad , \quad X - 3\sigma_\varepsilon) \quad (21)$$

are respectively the 95% , 99% and 99.73% confidence intervals of the parameter μ.

These respectively mean that

(i) out of 100 random intervals corresponding to 100 random observations (or 100 random samples), the value of μ will fall outside a maximum 5 such intervals defined by the equation (19) ,

(ii) out of 100 random intervals corresponding to 100 random observations (or 100 random samples), the value of μ will fall outside a maximum 1 such interval defined by the equation (20)

& (iii) out of 10000 random intervals corresponding to 10000 random observations (or 10000 random samples), the value of μ will fall outside a maximum 27 such intervals defined by equation (21).

4.3. Confidence Interval of the Observation Variable 'X '

Again under the assumption numbered (9),

$$X - \mu \sim N(0 , \sigma_\varepsilon).$$

This implies that $X \sim N(\mu , \sigma_\varepsilon)$.

Thus by the same area property of Gaussian distribution mentioned above,

$$P(\mu - 1.96\, \sigma_\varepsilon < X < \mu - 1.96\, \sigma_\varepsilon) = 0.95, \quad (22)$$

$$P(\mu - 2.58\, \sigma_\varepsilon < X < \mu - 2.58\, \sigma_\varepsilon) = 0.99 \quad (23)$$

$$\& \ P(\mu - 3\sigma_\varepsilon < X < \mu - 3\sigma_\varepsilon) = 0.9973 \quad (24)$$

Theses imply the intervals

$$(\mu - 1.96\, \sigma_\varepsilon \ , \ \mu - 1.96\, \sigma_\varepsilon) , \quad (25)$$

$$(\mu - 2.58\, \sigma_\varepsilon \ , \ \mu - 2.58\, \sigma_\varepsilon) \quad (26)$$

$$\& \ (\mu - 3\sigma_\varepsilon \ , \ \mu - 3\sigma_\varepsilon) \quad (27)$$

are respectively the 95% , 99% and 99.73% confidence intervals of the variable *X*.

These respectively mean that

(i) out of 100 random observations, maximum 5 observations fall outside the interval given by the equation (25),

(ii) out of 100 random observations, maximum 1 observation falls outside the interval given by equation (26)

& (iii) out of 10000 random observations, maximum 27 observations fall outside the interval given by equation (27)

4.4. Least Squares Estimator (LSE) of the Parameter μ

The LSE μ is obtained by minimizing

$$\Sigma\, \varepsilon_i^2 = \sum_{i=1}^{n} (Y_i - \mu)^2$$

with respect to μ.

The said estimator is thus

$$\hat{\mu} = (1/n) \sum_{i=1}^{n} Y_i \quad (28)$$

4.5. Maximum Likelihood Estimator (MLE) of the Parameter μ

The likelihood function of the observations

$$Y_1, Y_2, \ldots\ldots\ldots, Y_n$$

is given by

$$L(Y_1, Y_2, \ldots\ldots, Y_n : \mu, \sigma) = \prod_{i=1}^{n} \{\sigma (2\Pi)^{1/2}\}^{-1} . \, exp \, [- \tfrac{1}{2} \{(x - \mu)/\sigma\}^2]$$

which gives

$$log_e L(Y_1, Y_2, \ldots\ldots, Y_n : \mu, \sigma) = \sum_{i=1}^{n} \{\sigma (2\Pi)^{1/2}\}^{-1} . \, exp \, [- \tfrac{1}{2} \{(x - \mu)/\sigma\}^2]$$

Maximizing this with respect to μ its MLE is found to be

$$\hat{\mu} = (1/n) \sum_{i=1}^{n} Y_i \qquad (29)$$

4.6. Error in Estimator of μ

Since $\qquad Y_i = \mu + \varepsilon_i$

therefore, $\qquad \hat{\mu} = (1/n) \sum_{i=1}^{n} Y_i = \mu + (1/n) \sum_{i=1}^{n} \varepsilon_i$

This means, the estimator (both LSE and MLE) of μ suffers from an error

$$(1/n) \sum_{i=1}^{n} \varepsilon_i \qquad (30)$$

which may not be 0 (zero).

4.7. Estimator of Error 'ε'

Since $\qquad Y_i = \mu + \varepsilon_i$

& $\qquad \hat{\mu} = (1/n) \sum_{i=1}^{n} Y_i$

therefore, the estimator (both LSE and MLE) of error ε_i associated to the observation Y_i is

$$\hat{\varepsilon}_i = Y_i - \hat{\mu} = Y_i - (1/n) \sum_{i=1}^{n} Y_i \qquad (31)$$

5. Analysis of Annual Maximum Temperature at Guwahati

The annual maximum temperature at a location satisfies the Situation – 1 if the change in that is free from assignable cause(s). Assuming that the change in temperature at Guwahati is free from assignable cause, the observations on the annual maximum temperature at Guwahati has been analyzed in order to obtain various estimates possible. Values of annual maximum Temperature at Guwahati observed during the period from 1969 to 2010 have been presented in Table – II. Various estimates, obtained, have been presented in Table – III, Table – IV and Table –V.

Table – II
Observed values of Annual Maximum Temperature at Guwahati (in Degree Celsius)

Year	Observed value	Year	Observed value	Year	Observed value	Year	Observed value
1969	37.1	1979	38.6	1989	36.7	2002	35.7
1970	36.6	1980	35.1	1990	36.0	2003	37.4
1971	36.0	1981	35.8	1991	37.4	2004	38.0
1972	35.7	1982	36.5	1992	39.4	2005	36.6
1973	39.0	1983	36.7	1993	36.4	2006	38.0
1974	36.1	1984	37.2	1994	37.3	2007	37.3
1975	39.2	1985	36.5	1995	36.3	2008	37.3
1976	39.0	1986	38.4	1996	37.2	2009	38.0
1977	35.3	1987	37.2	2000	37.5	2010	37.2
1978	36.8	1988	36.3	2001	36.7	2011	–

Table – III
Estimated Value of Annual Maximum Temperature at Guwahati (in Degree Celsius)

Serial No.	Estimator	Estimate
1	Mean	37.0641
2	Standard Deviation	1.0569
3	95 % Confidence Interval	(34.99245 , 39.13575)
4	99 % Confidence Interval	(34.33713 , 39.79107)
5	99.73 % Confidence Interval (Natural Interval)	(33.89321 , 40.23499)
6	Number of Observations falling out side of the Corresponding Natural Intervals	0

Table – IV
Estimate of Chance Error (in Degree Celsius)

Serial No.	Parameter related to Chance Error	Estimate
1	Mean	– 0.03241
2	Standard Deviation	1.0569
3	95 % Confidence Interval	(– 2.07165 , 2.07165)
4	99 % Confidence Interval	(– 2.72697 , 2.72697)
5	99.73 % Confidence Interval (Natural Interval)	(– 3.1709 , 3.1709)
6	Number of Observations falling out side of the Corresponding Natural Intervals	0

Table – V
Estimate of Chance Error associated to Observation (in Degree Celsius)

Year	Annual Maximum Temperature	Estimate of Error	Year	Annual Maximum Temperature	Estimate of Error
1969	37.1	0.0359	1989	36.7	- 0.3641
1970	36.6	– 0.4641	1990	36.0	-1.0641
1971	36.0	– 1.0641	1991	37.4	0.3359
1972	35.7	– 1.3641	1992	39.4	2.3359
1973	39.0	1.9359	1993	36.4	- 0.6641
1974	36.1	- 0.9641	1994	37.3	0.2359
1975	39.2	2.1359	1995	36.3	- 0.7641
1976	39.0	1.9359	1996	37.2	0.1359
1977	35.3	- 1.7641	2000	37.5	0.4359
1978	36.8	- 0.2641	2001	36.7	- 0.3641
1979	38.6	1.5359	2002	35.7	-1.3641
1980	35.1	-1.9641	2003	37.4	0.3359
1981	35.8	-1.2641	2004	38.0	0.9359
1982	36.5	- 0.5641	2005	36.6	- 0.4641
1983	36.7	-0.3641	2006	38.0	0.9359
1984	37.2	0.1359	2007	37.3	0.2359
1985	36.5	- 0.5641	2008	37.3	0.2359
1986	38.4	1.3359	2009	38.0	0.9359
1987	37.2	0.1359	2010	37.2	0.1359
1988	36.3	- 0.7641			

6. Conclusion

1. In this study, analysis of error has been developed for the situation–1 only. Analysis of error is required to be attempted for each of the other situations, where associations of errors have been identified, discussed in this paper.
2. The situation discussed here corresponds to theoretically known relationships between two variables. There is necessity for studying the associations of errors in the situations where more than two variables are related by known theoretical relationships.
3. There is also necessity for studying the associations of errors in the situations where two and /or more variables are related by unknown theoretical relationships.

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